# Correlation between Nuclear Symmetry Energy parameters at saturation and subsaturation densities

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## Introduction

Accurate knowledge of the equation of state (EoS) of neutron-rich nuclear matter is essential to understand different aspects of finite nuclei and neutron stars. An important ingredient in the equation of state of asymmetric nuclear matter is the nuclear symmetry energy (NSE). The density dependence of NSE at saturation is more or less known to certain extent. However, at a density away from saturation, the behaviour of NSE is largely unknown. The density dependence of NSE can be understood through different parameters such as the density slope $L(\rho)$ and the curvature $K_{sym}(\rho)$ of nuclear symmetry energy $E_s(\rho)$ at saturation density $\rho_0$. These parameters of NSE appear in its expansion around the saturation density $\rho_0$

$$\begin{aligned} E_s(\rho) &= E_s(\rho_0) + L(\rho)\left(\frac{\rho-\rho_0}{3\rho_0}\right) \\ &+ \frac{K_{sym}(\rho)}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \cdots . \quad (1) \end{aligned}$$

The behaviour of NSE at a subsaturation reference density $\rho_c < \rho_0$ may be understood from the expansion of $E_s(\rho)$ around $E_s(\rho_c)$ as

$$E_s(\rho) = E_s(\rho_c) + L(\rho_c)\varepsilon + \frac{K_{sym}(\rho_c)}{2!}\varepsilon^2 + \mathcal{O}(\varepsilon^3), \quad (2)$$

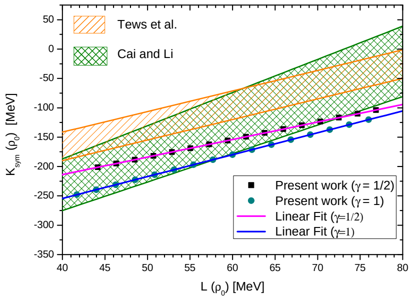

FIG. 1: $K_{sym}(\rho_0) - L(\rho_0)$ correlation obtained from the EoSs constructed using the finite range effective interaction (SEI). The correlation bands of Tews et al. [1] and Cai et al.[4] are also shown in the figure for comparison.

where $\varepsilon = \frac{\rho-\rho_c}{3\rho_c}$. Here we have defined the respective slope parameter and the curvature parameter of NSE at a reference density $\rho_c$ as

$$L(\rho_c) = 3\rho_c \frac{dE_s(\rho)}{d\rho}|_{\rho=\rho_c}, \quad (3)$$

$$K_{sym}(\rho_c) = 9\rho_c^2 \frac{d^2E_s(\rho)}{d\rho^2}|_{\rho=\rho_c}. \quad (4)$$

While there are constraints available in literature for the NSE parameters at saturation, reliable constraints on $L(\rho)$ and $K_{sym}(\rho)$ at a subsaturation density are not available [1, 2]. In the present work, we have tried to obtain a correlation between these parameters using density dependent finite range effective interactions.

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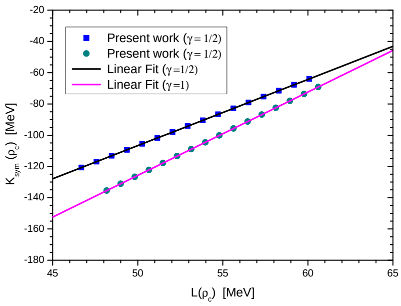

FIG. 2: $K_{sym}(\rho_c) - L(\rho_c)$ correlation.

## Formalism

We consider a finite range simple effective interaction (SEI)[3]

$$
\begin{aligned}
v_{eff}(\mathbf{r}) &= t_0(1+x_0P_\sigma)\delta(\mathbf{r}) \\
&+ \frac{1}{6}t_3(1+x_3P_\sigma)\left[\frac{\rho(\mathbf{R})}{1+b\rho(\mathbf{R})}\right]^\gamma \delta(\mathbf{r}) \\
&+ (W+BP_\sigma-HP_\tau-MP_\sigma P_\tau)\, f(r),
\end{aligned}
\tag{5}
$$

where $f(r)$ represents the form factor of a finite range Yukawa interaction $\frac{e^{-r/\alpha}}{r/\alpha}$, $\alpha$ being the range of the interaction. The parameter $\gamma$ decides the stiffness of the EoS. The parameters of SEI are fixed as per the procedure followed in Ref.[3]. We have constructed different EoSs for two values of $\gamma$ namely 1/2 and 1 corresponding to the incompressibility in nuclear matter $K = 240$ and 287 MeV. The respective parameters of the NSE are obtained both at saturation $\rho_0$ and subsaturation densities $\rho_c = 0.11$ fm$^{-3}$.

## Result and Discussion

In Fig. 1, we have correlated $K_{sym}(\rho_0)$ with $L(\rho_0)$ obtained from the constructed EoSs. Also, in the figure, the bands of the $K_{sym}(\rho_0) - L(\rho_0)$ correlation of Tews et al.[1] and Cai et al.[4] are shown. Our results lie below the correlation band of Tews et al. but are quite compatible with the correlation band of Cai et al. Linear fits are obtained for our $K_{sym}(\rho_0) - L(\rho_0)$ correlation which read as

$$
\begin{aligned}
K_{sym}(\rho_0) &= 2.98\ L(\rho_0) - 333.01, && (6)\\
K_{sym}(\rho_0) &= 3.72\ L(\rho_0) - 403.27. && (7)
\end{aligned}
$$

For a reasonable value of $L(\rho_0) = 60$ MeV, the above relation for $\gamma = 1/2$ yields $K_{sym}(\rho_0) = -154.09$ MeV.

There are no correlations available in literature for $K_{sym}(\rho_c) - L(\rho_c)$. In Fig. 2, we show the $K_{sym}(\rho_c) - L(\rho_c)$ correlation for our constructed EoSs. Linear fits to the results may be expressed respectively for $\gamma = 1/2$ and $\gamma = 1$ in the unit of MeV as

$$
\begin{aligned}
K_{sym}(\rho_c) &= 4.236\ L(\rho_c) - 318.50 \pm 0.03, \\
K_{sym}(\rho_c) &= 5.333\ L(\rho_c) - 392.38 \pm 0.004.
\end{aligned}
$$

## Conclusion

Even though the values of $E_s(\rho_0)$ and $L(\rho_0)$ are known to certain extent, there are no such constraints available for the curvature symmetry parameter $K_{sym}(\rho_0)$. So far, we have a crude information about $K_{sym}(\rho_0)$ as $-400 \leq K_{sym}(\rho_0) \leq -100$ MeV. In view of the role of $L(\rho_0)$ and $K_{sym}(\rho_0)$ in deciding the density dependence of nuclear symmetry energy and in obtaining the neutron skin thickness in finite nuclei to the radius of neutron stars, we wish to have a correlation between these two quantities as obtained from EoSs constructed using a finite range effective interaction. Also, we have obtained similar correlations at subsaturation densities. Our results compared well with some recently available results

## References

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