Parameter phase space in accordance with the very precise χEFT N^3LO equation of state

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Introduction

In this work we aim to study the uncertainty associated with different model parameters to construct equation of state (EOS) in the chiral effective mean field model with $\sigma-\rho$ and $\omega-\rho$ cross interactions in the meson-fields. Bayesian estimation of model parameter is calculated with minimal constraints based on nuclear saturation properties and low-density pure neutron matter EOS derived from a precise next-to-next-to-next-to-leading order (N^3LO) calculation in chiral effective field theory (χEFT).

Model and parameters

We have taken the Lagrangian for the effective chiral model which includes the various cross-coupling terms. Here, the interactions of ψ_B, the nucleon iso-spin doublet is via the (σ, ω, ρ) mesons and their cross-couplings $\sigma-\rho$ and $\omega-\rho$. The details of the model and its attributes can be found in Ref. [1].

Results

The chiral model parameters, namely C_σ, C_ω, B, C, C_ρ, η_1 and η_2 are evaluated within a Bayesian parameter estimation approach considering a minimal set of fit data related with the nuclear saturation properties and the pure neutron matter EOS obtained from a precise N^3LO calculation in χEFT . To get the marginalized posterior distributions of a given model parameter, within a Bayesian approach one simply needs a set of fit data, a theoretical model, and a set of priors for the model parameters. The joint posterior distributions of the model parameters are calculated with the help of the product of the likelihood function and the prior distributions. We take two minimal sets of fit data, referred here after as Case-I and Case-II sets. The only difference between these two cases is that we consider the uncertainties up to four times in case-II compared to case-I. The calculations are performed with Gaussian priors of all model parameters.

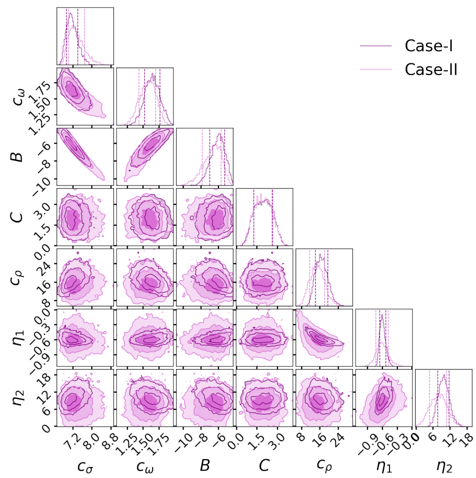

FIG. 1: Corner plots for the marginalized posterior distributions of the model parameters. One dimensional posterior distributions for both Case-I (purple) and Case-II (pink) are given along the diagonal plots. The vertical lines indicate the 68% confidence interval of the model parameters.

In Figure 1, we show the corner plots for the marginalized posterior distributions of the chiral model parameters C_σ, C_ω, B,C, C_ρ, η_1 and η_2 for both data sets Case-I and Case-II. Around eight thousand final sample parameters are obtained for both Case-I and Case-II. The diagonal plots in the figure compare the one dimensional marginalized posterior distribution of individual parameter for

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TABLE I: The median values of chiral model parameters, namely C_σ, C_ω, B, C, C_ρ, η_1 and η_2 along with 68% (90%) CI obtained for both data sets Case-I and Case-II are listed.

Para	Units	Case-I	Case-II
C_σ		$7.138^{+0.274(0.480)}_{-0.220(0.329)}$	$7.302^{+0.382(0.691)}_{-0.281(0.419)}$
C_ω	fm^{-2}	$1.644^{+0.121(0.201)}_{-0.120(0.197)}$	$1.577^{+0.125(0.198)}_{-0.141(0.221)}$
B		$-6.052^{+0.768(1.171)}_{-0.906(1.522)}$	$-6.598^{+0.896(1.346)}_{-1.211(2.043)}$
C	fm^{-4}	$1.982^{+0.688(1.040)}_{-0.698(1.028)}$	$1.937^{+0.687(1.080)}_{-0.670(0.988)}$
C_ρ	fm^{-2}	$16.606^{+2.898(4.886)}_{-2.605(3.921)}$	$14.790^{+3.249(5.687)}_{-3.256(5.000)}$
η_1		$-0.598^{+0.068(0.120)}_{-0.053(0.081)}$	$-0.610^{+0.132(0.240)}_{-0.118(0.200)}$
η_2		$9.690^{+2.025(3.612)}_{-2.003(3.088)}$	$7.918^{+2.979(4.981)}_{-3.134(4.935)}$

both Case-I and Case-II. The vertical lines indicates the 68% CI (Confidence input) of the model parameters. The elliptical nature of the 2D CI spells th correlations existing among those parameters, while the circular nature mean marginal/ no correlations. As one can be seen from the figure, the parameters C_σ - C_ω, C_σ - B as well as the parameters C_ω - B are noticeably correlated in both Case-I and Case-II, primarily due to the binding energy constraints applied to the fit data and their interactions within the model. The median value and 68% (90%) CI for all model parameters are listed in Table I for both Case-I and Case-II. In Case-II, the 90% CI for parameters C_σ, C_ω and B increased by $\sim (4-10)$ % compared to that of the Case-I. However, there are noticeable change in Case-II parameters compared to Case-I, particularly in the 90% CI for parameters C_ρ, η_1 and η_2, where the increase is $\sim (10-40)$% in Case-II.

We plot the symmetry energy as a function of density (ρ) with 90% CIs which are obtained from the posterior distributions of the parameters for both the cases. The spread we find in the 90% CI in Case-II is large about (3 times larger) as compared to Case-I. The lower extreme for both cases are similar, but throughout the density range 0.24 fm^{-3} and on-wards, the difference in the upper extreme for Case-II increases with density. It is to be noted that the symmetry energy component $J_{\mathrm{sym},0}$ is part of the fit data for both the cases. The constrained low density PNM EOS in Case-I reduces by 3 times compared to Case-II, up to highest density as shown. These results comply with the analysis in the original work done [2], where the parameters were estimated using the standard nuclear matter saturation properties. However, the effect of cross-couplings were not considered in the same.

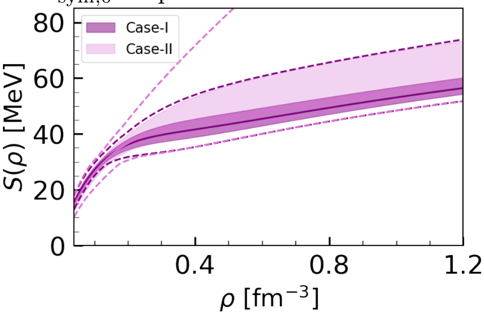

FIG. 2: The symmetry energy as a function of baryon density ρ for both cases Case-I (purple) and Case-II (pink). The dashed lines in both the panel represent the extremes.

Conclusions

We analyzed the uncertainty associated with the chiral model parameters using a Bayesian approach with minimal constraints. We conclude that the 90% CI for model parameters C_σ , C_ω and B in Case-II increases by $\sim (4-10)$% compared to Case-I, whereas it is $(10-40)$% increment in case of C_ρ, η_1 and η_2. It is also found that the symmetry energy component $J_{\mathrm{sym},0}$ is a part of the fit data for both the cases.

References

[1] N. K. Patra *et al.*, Astrophys. J. **900**, 1, 49 (2020).

[2] T. K. Jha and H. Mishra, Phys. Rev. C **78** (2008), 065802