Mass spectroscopy of charmonium using Cornell plus exponential potential

Sreelakshmi M* and Akhilesh Ranjan
Department of Physics,
Manipal Institute of Technology,
Manipal Academy of Higher Education,
576104, Manipal, Karnataka, India.

INTRODUCTION

Recently, there have been significant advancements in the study of heavy hadrons. Nonrelativistic potential models influenced by the fundamental QCD characteristics have been fairly successful in predicting a variety of hadron properties, including mass and decay. Quarkonia or heavy mesons ($Q\bar{Q}, Q = b, c$) which are bound states of a heavy quark and an antiquark, are generally thought to be non-relativistic ($m_{Q\bar{Q}} \gg \Lambda_{CD}$). As a result, calculating different features of these mesons has been extremely successful using a potential model. The search for the properties of heavy quarkonia give valuable insight into heavy quark dynamics.

Cornell potential is one of the extensively used potential for the hadrons [1]. The potential is a combination of Coulomb and linear terms. There will be one gluon exchange contribution at short distances, and at large distances, there is confinement of quarks [2], which takes care of the features of QCD. Lattice QCD calculations also support Cornell potential [3].

Here we have chosen Cornell potential combined with an exponential type potential of the form $\alpha r e^{-\beta r}$ to find out the mass spectrum of charmonium. It is expected that potential will help as an intermediate form to get a good fit with experimental data. This intermediate form is inspired by the Wisconsin potential [4].

* sreelakshmim.araam@gmail.com

FORMULATION

The non relativistic Hamiltonian can be written as [5],

$$H = M + \frac{p^2}{2m} + V(r),$$

$$M = m_Q + m_{\bar{Q}}, m = \frac{m_Q m_{\bar{Q}}}{m_Q + m_{\bar{Q}}},$$

here, p is the relative momentum of each quark, M is the total mass of quark and antiquark and m_Q is the mass of each quark and $V(r)$ is the quark-antiquark potential.

$$V(r) = V_0 + \frac{-\alpha_s}{r} + \sigma r + \alpha r e^{-\beta r},$$

where, α_s is strong coupling constant, σ is string tension, V_0 is a constant term associated with the self energy contribution [6]. α and β are constant potential parameters. The value of V_0 and σ is fitted with the experimental value of $1S$ state. The strong coupling constant can be determined from,

$$\alpha_s(q^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f)\ln\frac{q^2}{\Lambda^2}},$$

where, Λ is the QCD scale factor, which can be determined experimentally. The value of Λ is taken as $0.16\ GeV$.

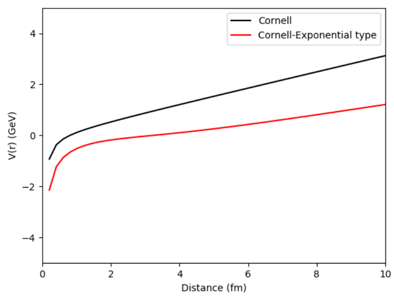
RESULTS AND DISCUSSION

The mass spectra of quarkonia is calculated using the formula,

$$M = 2m_Q + E_{nl}.$$

The Schroedinger equation is solved numerically with the help of the Mathematica package [7]. Cornell potential and Cornell combined with exponential type potential is plotted in FIG 1.

FIG. 1. Plot of Cornell and Cornell plus exponential type potential.



There are six parameters in the potential including α_s, σ, α, β, V_0 and the charm quark mass m_c. The quark mass is chosen as 1.27 GeV [8]. The results are obtained and compared with experimental and other theoretical studies in Table I.

TABLE I. Mass spectra of charmonium in GeV. ($\alpha_s = 0.31428$, $\sigma = 0.202\ GeV^2$, $\alpha = 0.80\ GeV^2$, $\beta = 0.75\ GeV$, $V_0 = 0.783\ GeV$ and m_c=1.27 GeV)

State	Present work	[9]	[10]	[11]	Exp.[8]
1S	3.096	3.096	3.068	3.096	3.096
1P	3.359	3.255	3.526	3.295	3.414
1D	3.581	3.504	3.829	3.583	
2S	3.524	3.686	3.697	3.686	3.637
2P	3.756	3.779	3.993	3.802	
2D	3.962			3.976	
3S	3.914	4.040	4.144	4.040	
4S	4.277	4.269		4.269	
5S	4.617	4.425			

CONCLUSION

The combination of Cornell plus exponential potential reproduces the mass spectra of charmonium. As we go to the higher order states, the results are getting closer to experimental and other theoretical studies. It is expected that, adding spin-dependent interactions may provide better results.

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