Mass spectra in radial excited state for (P-wave) Charmonia and Bottomonia

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Introduction

Quarkonia ($Q\bar{Q}$) are bounded with heavy(Q) quark and antiquark($\bar{Q}$) [1]. Many X Y Z resonance states has been discovered in last few years and are still examination finding in relativistic and non relativistic potential model [2]. The success of various of theoretical (phenomenological) model like relativistic quark model [3], Non- relativistic quark model [2, 4–7] are help us for more calculation of Quarkonia. Recently reported by LHCb and BELLE new bottomoia states $\chi_{b1}(1P)$, $Z_b(10610)$ and $Z_b(10650)$ respectively [8, 9]. In Charmonia $X(4274)$ is suggested as a 3^3P_1 state for P wave as a admixture [10]. The same prediction above both states of in our model by P sates. For short distance running coupling constant (α_s) is important parameter in heavy quark (m_Q) while for long distant non relativistic function ($\mathcal{O}$ $(\frac{1}{m})$) or its derivative. Apart from all potential we have chosen screening potential $V_s(r) = \frac{A}{\mu}(1 - e^{-\mu r})$ in calculation of mass spectra of Quarkonia for better improvisation.

Spin-Spin splitting in ($Q\bar{Q}$)

The phenomenological potential well working in light-heavy ($q\bar{Q}$) and heavy-heavy ($Q\bar{Q}$) flavour mesons. Hence, for the study of the bound states of mesons, we apply semi-relativistic Hamiltonian [2].

$$H = \sqrt{p^2 + m_Q^2} + \sqrt{p^2 + m_{\bar{q}}^2} + V(\tilde{r}); \quad (1)$$

here, p is the relative momentum of the quark anti-quark, m_q and $m_{\bar{Q}}$ are light (up/down) and heavy (charm/beauty) quark masses. For spin-independent port, we have chosen below potential for calculations [12–15]

$$V(\vec{r}) = -\frac{4}{3}\frac{\alpha_s}{r} + \frac{A}{\mu}(1 - e^{-\mu r}) + V_0 \quad (2)$$

Here, we utilize the Ritz variational strategy for the study of the mesons. In $Q\bar{Q}$ mesons, the confining interaction plays an important role. We employ a Gaussian wave function, to predict the expectation values of the Hamiltonian. These wave functions are taken from the solution of the relativistic Hamiltonian, as described in Ref.[2] (and references therein). Using the virial theorem, the variational parameter, μ is calculated for each state [11].

$$\langle K.E\rangle = \frac{1}{2}\left\langle \frac{rdV}{dr} \right\rangle \quad (3)$$

For P state, the spin-dependent potential for calculating spin-spin interaction is given and perturbatively added [16].

$$V_{spin}(r) = \frac{4}{3}\alpha_S \frac{2}{3m_1 m_{\bar{2}}} S_1 \cdot S_{\bar{2}} 4\pi\delta(r) \quad (4)$$

Results and Discussion

The Gaussian wave function is used to determine the masses of Quarkonia in P state. Using screened potential we observed that effects are applying when, "μ" is the screening factor that makes the long-range

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TABLE I: Mass spectra of $c\bar{c}$ and $b\bar{b}$ meson in excited states (in GeV).

$n^{2S+1}L_J$	J^{PC}	Recent study	PDG[1]	Ref.[2]	Ref.[3]	Ref.[4]	Ref.[7]
1^3P_0	0^{++}	3.492	3.414±0.031 ($\chi_{c0}(1P)$)	3.457	3.413	3.398	3.473
1^3P_1	1^{++}	3.517	3.510±0.007 ($\chi_{c1}(1P)$)	3.523	3.511	3.424	3.506
1^1P_1	1^{+-}	3.543	3.525±0.038 ($h_c(1P)$)	3.534	3.525	3.450	3.527
1^3P_2	2^{++}	3.553	3.556±0.007 ($\chi_{c2}(1P)$)	3.556	3.555	3.552	3.511
1^3P_0	0^{++}	9.877	9.859±0.004 ($\chi_{b0}(1P)$)	9.874	9.859	9.755	9.837
1^3P_1	1^{++}	9.889	9.892 ±0.013 ($\chi_{b1}(1P)$)	9.894	9.892	9.768	9.852
1^1P_1	1^{+-}	9.902	9.899±0.008 ($h_b(1P)$)	9.899	9.900	9.775	9.864
1^3P_2	2^{++}	9.908	9.912±0.013 ($\chi_{b2}(1P)$)	9.907	9.912	9.792	9.877

scalar potential of $V(r)$ behave like Ar when $r \ll 1/\mu$, and becomes a constant A/μ when $r \gg 1/\mu$. The screened potential shrinks the masses of higher excited states. This study shows the success of relativistic correction with the potential model for a spectroscopic notation for $Q\bar{Q}$. Using Eqn.(4) we generalised bound sate of P state J^{PC} values. In P wave ($L = 1, S = 0$) $J^{PC} = 1^{+-}$ while ($L = 1, S = 1$) $J^{PC} = 0^{++}, 1^{++}, 2^{++}$. In P sate ($L = 1, S = 0$) $J^{PC} = 1^{+-}$ represent center weight masses and its spin-orbit contribution becomes zero, while ($L = 1, S = 1$) for J^{PC} represent mixes states singlet and triplet.

In $c\bar{c}$ for P state it can be seen that 1^3P_0, 1^3P_1, 1^1P_1 and 1^3P_2 estimated mass difference between states are differ only by 0.025, 0.026, 0.01 and 0.061 GeV respectively. Compare with experiment masses (0.096, 0.015, 0.031 and 0.142 GeV) our estimation are slightly decreased [1]. 1^3P_1 and 1^3P_2 states are good agreement with PDG [1] so it is assign as a newly observed charmonium states $\chi_{c1}(3511)$ and $\chi_{c2}(3556)$. Comparing $b\bar{b}$ with PDG [1], we observed 1P state varies with range 0.1 to 0.3 %. In 1P state for 1^3P_1, 1^1P_1 and 1^3P_2 are candidate for $\chi_{b1}(9892)$, $h_b(9899)$ and $\chi_{b2}(9912)$ for $b\bar{b}$ with their respective Quantum number $J^{PC} = 1^{++}, 1^{-+}, 2^{++}$. In 1P state only 0.018, 0.003, 0.003 and 0.004 GeV differ from the PDG [1] and good matched with other references [2, 3].

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