Generalized TMDs of quark at non-zero skewness

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Introduction

Generalized TMDs (GTMDs) have been defined in the literature and used to study the spin and orbital angular momentum of partons. GTMDs are a generalized version of well known TMDs (examples: Siver and Boer-Mulder function), which has been used extensively to study the parton in a bound state. There are 16 GTMDs for quarks at the twist-2 level. Out of these 16, two GTMDs known in the literature as $F_{1,4}$, and $G_{1,1}$ are most significant as both are related to the orbital angular momentum (OAM) and the spin of the partons.

GTMDs Definition

The GTMDs have been calculated for zero skewness due to the mathematical simplicity. Though, the zero-skewness greatly reduces the mathematical complexity, in the experiment, skewness is never zero. This makes it interesting and compelling case to evaluate the same for non-zero skewness. The GTMDs are defined via bilinear decomposition of quark-quark correlator [1]

$$W^{[\gamma^+]}_{\lambda,\lambda'} = \frac{1}{2m}\bar{u}(p',\lambda')\Big[F_{1,1} - \frac{i\sigma^{i+}k_{i\perp}}{P^+}F_{1,2} - \frac{i\sigma^{i+}\Delta_{i\perp}}{P^+}F_{1,3} + \frac{i\sigma^{ij}k_{i\perp}\Delta_{j\perp}}{m^2}F_{1,4}\Big]u(p,\lambda).$$

The quark-quark correlator $W^{[\Gamma]}_{\lambda,\lambda'}(x,\xi,\mathbf{k}_\perp,\mathbf{\Delta}_\perp)$ is defined through non-diagonal matrix element of the bi-local quark field,

$$W^{[\Gamma]}_{\lambda,\lambda'}(x,\xi,\mathbf{k}_\perp,\mathbf{\Delta}_\perp) = \frac{1}{2}\int\frac{dz^-}{2\pi}\frac{d^2z_\perp}{(2\pi)^2}e^{ip\cdot z} \Big\langle p',\lambda'\Big|\overline{\psi}(-z/2)\mathcal{W}_{[-z/2,z/2]}\Gamma\psi(z/2)\Big|p,\lambda\Big\rangle\Big|_{z^+=0}.$$

The state $|p,\lambda\rangle$ and $|p',\lambda'\rangle$ are the initial and final states of dressed quark. $\mathcal{W}_{[-z/2,z/2]}$ is the color gauge link and reduces to unity in the light-front gauge. Γ is an element of the set $\{\gamma^+,\gamma^+\gamma^5, i\sigma^{+j}\gamma^5\}$ and is chosen depending on the polarization of quarks.

Dressed Quark Model

The dressed quark can be considered as a bound state of a quark and a gluon. A dressed quark state with momentum p and helicity λ can be expanded as [2]

$$\Big|p^+,p_\perp,\sigma\Big\rangle = \Phi^\sigma(p)b^\dagger_\sigma(p)|0\rangle + \sum_{\sigma_1\sigma_2}\int[dp_1]\int[dp_2]\sqrt{16\pi^3p^+}\delta^3(p-p_1-p_2)\Phi^\sigma_{\sigma_1\sigma_2}(p;p_1,p_2)b^\dagger_{\sigma_1}(p_1)a^\dagger_{\sigma_2}(p_2)|0\rangle;$$

where $[dp] = \frac{dp^+d^2p_\perp}{\sqrt{16\pi^3p^+}}$. $b^\dagger(a^\dagger)$ is creation operator for quark(gluon), and $\Phi^\sigma(p)$, $\Phi^\sigma_{\sigma_1\sigma_2}$ represents single particle and two-particle wave function respectively.

Results and Discussions

Choosing a symmetric frame for kinematics such that the initial and final momentum of dressed quark is given by

$$p = \left((1+\xi)p^+, \Delta_\perp/2, \frac{m^2+\Delta_\perp^2/4}{(1+\xi)p^+}\right);$$
$$p' = \left((1-\xi)p^+, -\Delta_\perp/2, \frac{m^2+\Delta_\perp^2/4}{(1-\xi)p^+}\right),$$

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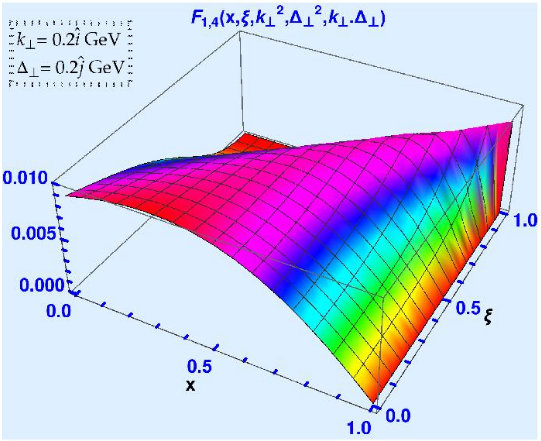

FIG. 1: Plot of GTMD $F_{1,4}$ as a function of x, and ξ.

where the four-momentum transfer from the target state is

$$\Delta = p' - p = \left(2\xi p^+, \Delta_\perp, \frac{t + \Delta_\perp^2}{2\xi p^+}\right),$$
$$\text{and} \quad t = -\frac{4\xi^2 m^2 + \Delta_\perp^2}{1 - \xi^2}.$$

The analytical expression for the GTMDs $F_{1,4}$ in the model comes out to be

$$F_{1,4} = \frac{\alpha}{(1-x)}\Big[2m^2(1+x)(1-\xi^2)\Big],$$

where

$$\alpha = \frac{N}{D(q_\perp, y)D^*(q'_\perp, x')(x^2 - \xi^2)},$$
$$D(k_\perp, x) = \left(m^2 - \frac{m^2 + k_\perp^2}{x} - \frac{k_\perp^2}{1-x}\right),$$

and $N = \frac{g^2 C_f}{2(2\pi)^3}$, g is the strong coupling constant, and C_f is the color factor. The analytical form of all other GTMDs will be presented during the symposium. However, this ξ dependence of $F_{1,4}$ does not affect the orbital angular momentum and spin-orbit correlation of quarks as both are defined for GTMDs in the limit of $\xi \to 0$. Since the expression of $F_{1,4}$ in limit $\xi \to 0$ agrees with [3], the OAM for quarks which are defined as [4, 5]

$$l_z^q = -\int dx d^2k_\perp \frac{k_\perp^2}{m^2} F_{14}$$
$$= -2N \int dx(x^2 - 1)\Big[I_1 - m^2(x-1)^2 I_2\Big],$$

also agrees with the results of [2, 3].

Conclusion

The understanding of ξ (skewness) dependence of GTMDs is significant if we aim to extract it from the experiment as skewness is never zero in an experiment. We derived the ξ dependence of GTMD $F_{1,4}$ here as it has attracted much attention due to its relation with the OAM of partons. The expression derived for $F_{1,4}$ agrees with [3] in the limit $\xi \to 0$. More interesting results of other GTMDs will be presented in the symposium. The study of ξ dependence of $F_{1,4}$ is a part of our broader work on GTMDs, which will be communicated soon for publication.

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