Fractality and phase transition studies for π^+ and π^- in $p-p$ collisions at $\sqrt{s} = 13$ TeV

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Introduction

From relativistic heavy ion collisions under extreme conditions of temperature and/or density, we get the valuable informations about the the spatiotemporal progression of the multiparticle production process. The existence of QGP like state of nuclear matter for nucleus–nucleus collisions at relativistic and ultra-relativistic energies was confirmed by the experiment at SPS at CERN, the RHIC at BNL and LHC at CERN. Now, $p-p$ collisions can be considered as the promissing probe to know the underlying dynamics of the heavy-ion collisions. In this study we have used Ultra-relativistic Quantum molecular Dynamics (UrQMD) model [1] to generate the events in $p-p$ collisions and studied the different properties of charged particle multiplicities. It is also very true that the UrQMD simulation has gained huge popularity to generate the events in $p-p$ collisions. The creation of pions for $p-p$ collisions were investigated through the mean multiplicity, rapidity spectra and particle ratios as a function of collision energy. We have considered the UrQMD model produced data to observe the intermittency type particle dynamical fluctuations of π^+ and π^- separately for $p-p$ collisions, at $\sqrt{s}$ =13 TeV energy in the form of scale factorial moment. To justify the utility of UrQMD model, we have compared p_T spectra of pions for UrQMD events with ALICE experimental data for $p-p$ collisions at $\sqrt{s} = 13$ TeV [2] .

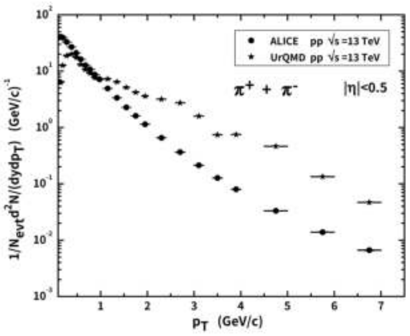

FIG. 1: Comparison of the p_T spectra of pions between the ALICE data and the UrQMD simulated data for p-p interactions at $\sqrt{s} = 13\,TeV$.

Method of Analysis

According to Bialas and Peschanski [3] in one–dimensional phase–space which is divided into M number of bins, the factorial moment F_q of order q will be given by
$F_q = M^{q-1} \sum_{m=1}^{M} \frac{\langle n_m(n_m-1)....(n_m-q+1)\rangle}{n_m^q}$,
where n_m represents the number of net pions (i.e. sum of all $\pi^+ or\ \pi^-$) in m^{th} bin and q is the order of moment. Here the bracket $\langle \dots \rangle$ represents the event average.

The anomalous fractal dimension "d_q" on intermittency exponent "α_q", can be written as $d_q = \frac{\alpha_q}{q-1}$, where d_q is also known as Renyi Co-dimension. Now the general fractal dimension "D_q" is also given by the equation $D_q = 1 - d_q$.
In order to find the degree of multifractality, we have consider the following eqn.
$\frac{d_q}{d_2} = (1-r) + \frac{qr}{2}$,
where "r" implies the degree of multifractality of the colliding particles. A non-zero values of "r" indicate the presence of multifractal behaviour [4] . The existence of non–thermal

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TABLE I: Value of different parameters depending on the scale factorial moment analysis in one dimensional η space for π^+ & π^-

Parameters	π^+	π^-
r	0.622±0.064	0.786±0.066
ν	1.507±0.046	1.582±0.047
μ	1.630±0.104	1.798±0.016
c	0.057±0.037	0.118±0.036

phase transitions can be verified through a function λ_q, where $\lambda_q = \frac{\alpha_q+1}{q}$. In order to observe the phase transitions, a function β_q is defined as $\beta_q = \frac{d_q}{d_2}(q-1)$. Ginzburg and Landau have also derived a critical exponent ν, which can be calculated by the equation $\beta_q = (q-1)^\nu$. According to Levy law, the ratio of anomalous dimension depends on μ through the relation $\beta_q = \left(\frac{q^\mu - q}{2^\mu - 2}\right)$. Now the multifractal specific heat (c) can be derived through the eqn. $D_q = (a-c) + c\left(\frac{\ln q}{q-1}\right)$, where "$a$" is a constant and "$c$" indicates the transition between multifractality and monofractality for relativistic heavy ion collisions.

Discussions

We have estimated the values of factorial moment for different order q in one dimensional η space using the relation $\langle F_q \rangle \propto M^{\alpha_q}$ which are shown in Fig. 2. We get the intermittency exponent α_q for different order q from the best fitted linear behaviour curve in Fig. 2 for π^+ & π^- separately. We have derived the values of "r", "ν", "μ" & "c" by using the values of anomalous fractal dimension α_q. From the variation of β_q vs $(q-1)$ we have derived the value of critical exponent "ν" and we have observed that for one dimensional η space some of the values of "ν" are close to 1.304 and some are not which in accordance to the framework of GL theory agrees quark-hadron phase transitions may exists for the values close to 1.304 and remaining values of "ν" doesn't as they are not close to the given value of "ν". It has been observed that π^- is more multifractal than π^+.

From the D_q vs $(\frac{lnq}{q-1})$ graph we have de-

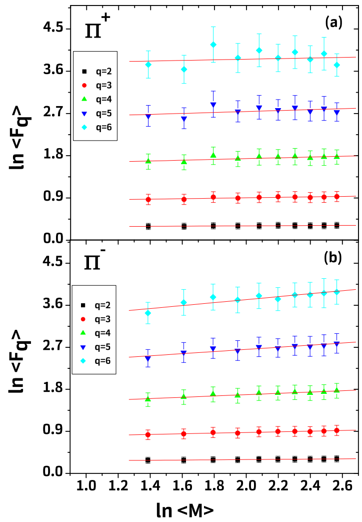

FIG. 2: Variations of $ln < F_q >$ with respect to lnM in one–dimensional η space (a) for π^+ and (b) π^-

rived the value of multifractal specific heat (c). For 1D η space, the value of "c" for π^+ and π^- being greater than zero which signals the presence of the transformations from the multifractality to monofractality for p-p collisions at $\sqrt{s}$=13 TeV.

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References

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