A consistent quasiparticle approach to QCD thermodynamics

Pracheta Singha[1],* Chowdhury Aminul Islam[2], Munshi G. Mustafa[3], and Rajarshi Ray[4]

[1] *National Institute of Science Education and Research*
[2] *School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China*
[3] *Theory Division, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata 700064, India and*
[4] *Center for Astroparticle Physics & Space Science, Bose Institute, Block-EN, Sector-V, Salt Lake, Kolkata-700091, India*

Introduction

The expectation value of the traced Polyakov loop $(\Phi, \bar{\Phi})$ is associated to the order parameter for the confinement-deconfinement phase transition of the pure gauge system. In the Polyakov extended chiral effective QCD models, quarks are considered to be coupled to these constant background Polyakov fields and the gluonic contribution to the medium thermodynamics is modeled via a Landau polynomial potential, $U(\Phi, \bar{\Phi})$. As an alternative to $U(\Phi, \bar{\Phi})$, a gluonic quasiparticle description is often discussed ([1] and refs there in), where the dynamical information for gluonic degrees of freedom can also be accessed. However these studies within the saddle point approximation have unphysical outcomes at low temperatures. Here, we will represent a novel approach to include the gluonic quasiparticles to the chiral Nambu—Jona-Lasinio (NJL) model and achieve physically consistent quasiparticle description of the QCD thermodynamics.

Formulation

Consider the thermodynamic potential,

$$\Omega[\theta_1(\mathbf{x}), \theta_2(\mathbf{x}), \sigma(x)] = \Omega_{\rm q}[\theta_1(\mathbf{x}), \theta_2(\mathbf{x}), \sigma(x)] + \Omega_{\rm g}[\theta_1(\mathbf{x}), \theta_2(\mathbf{x})] \ , \quad (1)$$

where, σ is the order parameter for chiral phase transition and θ_1, θ_2 are the SU(3) class parameters such that,

$$\Phi = \frac{1}{3}\mathrm{Tr}(e^{i\theta_1}, e^{i\theta_2}, e^{-i(\theta_1+\theta_2)}) \quad (2)$$

one can write,

$$\frac{\Omega_{\rm g}}{T} = 2\int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \sum_{n=1}^{8} a_n e^{-\frac{n\epsilon_g}{T}}\right) , \quad (3)$$

where, the coefficients $a_n(\Phi, \bar{\Phi})$ are polynomial functions of $\Phi, \bar{\Phi}$ [2] and $\epsilon = \sqrt{p^2 + m_g^2}$ is the energy corresponding to gluon quasiparticles. Ω_q is the quark thermodynamic potential for 2 flavour system in the usual PNJL model,

$$\Omega_{\rm q} = \frac{\sigma^2}{2G} - 12\int_0^\Lambda \varepsilon_p - \int \frac{d^3p}{(2\pi)^3} \ln\left[[1 + e^{-3\beta\varepsilon_p} + 3(\Phi + \bar{\Phi}e^{-\beta\varepsilon_p})e^{-\beta\varepsilon_p}][1 + e^{-3\beta\varepsilon_p} + N_c(\bar{\Phi} + \Phi e^{-\beta\varepsilon_p})e^{-\beta\varepsilon_p}]\right] , \quad (4)$$

where $\epsilon_p = \sqrt{p^2 + (m_0 + \sigma)^2}$. In Ref. [2] for pure gauge sector we had the partition function as,

$$Z_{\rm g} = \int \mathcal{D}\theta_1 \mathcal{D}\theta_2 \exp\left[-\frac{1}{T}\int d^3x \Omega_{\rm g}[\theta_1(\mathbf{x}), \theta_2(\mathbf{x})]\right] = \int \prod_{\mathbf{x}} \frac{1}{24\pi^2} d\theta_1(\mathbf{x}) d\theta_2(\mathbf{x}) \mathrm{H} \exp\left[-\frac{1}{T}\int d^3x \Omega_{\rm g}[\theta_1(\mathbf{x}), \theta_2(\mathbf{x})]\right] , \quad (5)$$

where, the Vandermonde determinant H is,

$$\frac{\mathrm{H}}{27} = [1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2]. \quad (6)$$

*Electronic address: `pracheta.singha@gmail.com`

In the presence of quarks, $\Omega_g \to \Omega$. If we replace $\sigma(x)$ by its saddle point solution, σ_{mf}, we can write the partition function as, $Z = z^N$ where,

$$z = \int \frac{d\theta_1 d\theta_2}{24\pi^2} \mathrm{H} \exp\left[-\frac{v}{T}\Omega[\theta_1, \theta_2, \sigma_{\rm mf}]\right] \quad (7)$$

Here we have assumed that the configuration space can be split into N $\to \infty$ equivalent and independent points as the thermodynamic potential contains no spatial derivatives [2]. Now one can evaluate the thermodynamic quantities starting from the relation,

$$p = \frac{T}{V} \ln Z = \frac{T}{v} \ln z \quad (8)$$

and the expectation values of local operators as,

$$< O[\Phi, \bar{\Phi}] >= \frac{1}{z} \int \frac{d\theta_1 d\theta_2}{24\pi^2} \mathrm{H}\, O[\Phi, \bar{\Phi}] \exp\left[-\frac{v}{T}\Omega[\theta_1, \theta_2, \sigma_{\rm mf}]\right] . \quad (9)$$

Results and discussion

To obtain numerical results the NJL parameters are taken from Ref. [1] and we took, $v = (2/T_d)^3$, with $T_d = 0.270$GeV. To mimic the crossover transition we have considered a continuous parametric form for gluon quasi-particle mass, as

$$m_g(T)/T = \alpha + \exp\left[-\beta\left(\frac{T}{T_d} - \gamma\right)\right] , \quad (10)$$

with the parameters $\alpha = 0.662961, \beta = 3.57009, \gamma = 1.13959$. In Fig. 1 we have shown the thermal variation of $\langle\Phi\rangle$ and the constituent quark mass as $\frac{m^*=m_0+\sigma}{m_0^*=m_0+\sigma(T=0)}$ which vary smoothly across the temperature range implying crossovers. The critical temperature T_c is obtained taking the average of two peak positions of the temperature derivatives of these expectation values, shown in the inset of Fig. 1. To show the physical consistency we have shown the thermal variation of pressure compared to the lattice data [3] in Fig. 2.Thus with this model framework we can obtain a consistent thermodynamic description of 2 flavour QCD system and can describe interactions that need dynamical information from both of the partonic sectors.

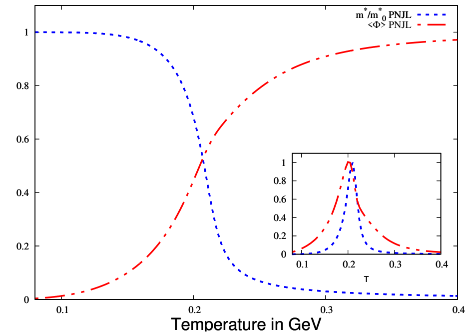

FIG. 1: Thermal variation of $\langle\Phi\rangle$ and constituent quark mass with their temperature derivatives.

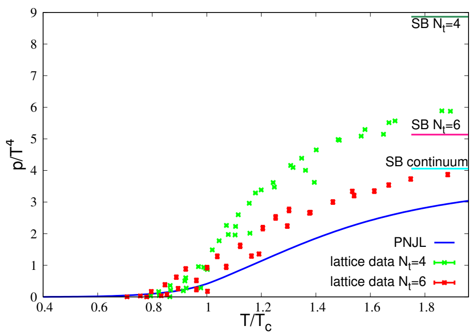

FIG. 2: Thermal variation of pressure compared with 2 flavour Lattice data [3].

Acknowledgments

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