Efficiency calibration of HPGe detectors using General Linear Model and Covariance Error Model

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Introduction

The calibration sources ^{133}Ba and ^{152}Eu consist of many characteristic γ-lines over a wide range of energy. These sources of known activity can be used for the efficiency calibration of high-purity germanium (HPGe) detectors. In nuclear reaction measurements, the knowledge of efficiency for characteristic γ-lines of reaction products aids in determining transition intensities as well as cross sections. Thus, the calibration of the detector efficiency is one of the most important aspects for the determination of many physical quantities in nuclear physics. In the present study, the efficiency has been calculated with help of two statistical models and compared with the experimentally observed values. Also, we have computed the uncertainties in efficiency which reduce significantly due to the use of correlation matrix. Finally, a comparison between the two models has been shown.

Definition and measurements

Efficiency of the HPGe detector is given by,

$$\epsilon = \frac{\text{Number of photons registered in the detector}}{\text{Number of photons emitted from the source}}$$

In terms of measurable quantities,

$$\epsilon_\gamma = \frac{c_\gamma}{A_0 e^{-\lambda t} I_\gamma}. \quad (1)$$

We have used a mixed radioactive source of ^{133}Ba and ^{152}Eu with 25 γ-transitions ranging from 81 keV to 1408-keV energy.

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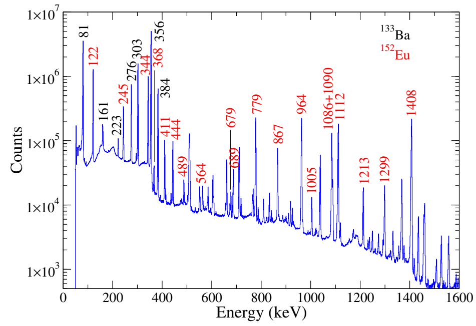

FIG. 1: The γ-energy spectrum obtained from the decay of ^{133}Ba and ^{152}Eu mixed source.

General Linear Model

Let us study a case from linear algebra where we aim at fitting a set of data $y = \{y_0, y_1, y_2, ...y_{n-1}\}$. We could think of these data as a result of an experiment and are functions of a series of variables $x = \{x_0, x_1, x_2, ...x_{n-1}\}$.

The simplest way is to parametrize our function in terms of a polynomial of degree $(m-1)$ with n points.

$$y(x) \to y_i(x_i) = \tilde{y_i} + \delta_i = \sum_{j=0}^{m-1} \beta_j x_i^j + \delta_i. \quad (2)$$

Using log transformation of these variables as,

$$y_i \to \ln(\epsilon_i) \text{ and } x_i \to \ln(E_{\gamma i}), \quad (3)$$

and vector-matrix notation, we get,

$$\ln(\boldsymbol{\epsilon}) = \boldsymbol{X\beta} + \delta', \quad (4)$$

where $\boldsymbol{X}$ is new design matrix of dimension $n \times m$, whose elements are $X_{ij} = [\ln(E_{\gamma_i})]^j$.

Instead of solving the linear algebra problem, we define a function, $C(\beta)$, which gives a measure of the spread between the exact values y_i and the parameterized values $\tilde{y}_i$,

$$C(\boldsymbol{\beta}) = \frac{1}{n}(\boldsymbol{y}-\tilde{\boldsymbol{y}})^T(\boldsymbol{y}-\tilde{\boldsymbol{y}}) = \frac{1}{n}(\boldsymbol{y}-\boldsymbol{X\beta})^T(\boldsymbol{y}-\boldsymbol{X\beta}) \tag{5}$$

In order to find the parameters β_j we will then minimize the spread of $C(\boldsymbol{\beta})$. Solving we get,

$$\boldsymbol{\beta} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y} \tag{6}$$

Resampling methods involve repeatedly drawing samples from a training set and refitting a model of interest. In order to estimate the variability of a linear regression fit, we can repeatedly draw different samples from the training data, fit a linear regression to each new sample, and then examine the extent to which the resulting fits differ. With 25 datapoints, We have used k-fold cross validation with $k = 5$. It divides all the samples into 5 groups of 5 datapoints. So that, in every resampling time, train data size is 80% of the total data and the test data is 20%.

Further, we have used the repetition for 20 times, producing different splits in each repetition. For scoring of the model, we have computed the mean squared error.

Covariance Error Model

We generalize the discussion of error propagation in the context of efficiency (ϵ_i and ϵ_j) of the detector with respect to i-th and j-th γ-lines. To measure covariance error between them, we introduce the concept of covariance matrix of total uncertainties (V_ϵ) and partial uncertainties ($E_qS_qE_q$) as,

$$V_\epsilon = \sum_{q=1}^{4} E_qS_qE_q \tag{7}$$

And finally, macro correlation matrix,

$$C_{\epsilon_{ij}} = \frac{V_{\epsilon_{ij}}}{\sqrt{V_{\epsilon_{ii}} \cdot V_{\epsilon_{jj}}}} \tag{8}$$

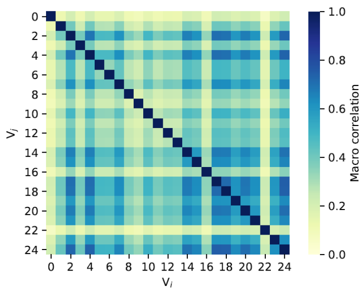

FIG. 2: Macro correlation matrix.

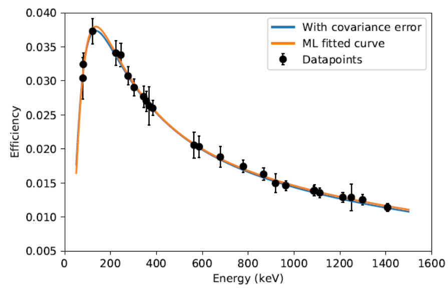

FIG. 3: A comparison of the efficiency predicted by the two models.

Results and Discussions

The energy depended efficiency calibration of HPGe detector corresponding to characteristic γ-energies of ^{133}Ba and ^{152}Eu has been carried out. The fitting parameters have been estimated by linear regression model and covariance error model. Both models predict the efficiency successfully. The maximum difference between the two models is around 2-3%.

Acknowledgements

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References

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