Investigation of the structure of even-even Ru nuclei using the IBM-1 framework

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Introduction

The Interacting Boson Model (IBM), proposed in 1974 [1], has been a crucial framework for studying nuclear structure. It explores quantum shape-phase transitions and the evolution of low-lying nuclear states based on proton and neutron numbers. In the IBM-1 variant, which does not differentiate between proton and neutron pairs, nuclear shapes are linked to SU(5), O(6), and SU(3) symmetries.

Nuclei are classified on the edges of a symmetry triangle, with vertices representing SU(5) (vibrator), O(6) (γ-soft), and SU(3) (symmetric rotor). The edges denote transitional regions, with a second-order phase transition along the SU(5)–O(6) edge and a first-order transition along the SU(5)–SU(3) edge. Most nuclei, including Os and Th isotopes, lie within the triangle rather than on its edges. Iachello [2] identified that critical points in this scheme correspond to the symmetries E(5) and X(5). At critical parameter values, energy levels are zeros of Bessel functions with half-integer and irrational indices. This paper investigates the structure of even-even Ru nuclei using the IBM-1 framework.

The Interacting Boson Model (IBM-1)

The Hamiltonian for calculating level energies has several equivalent forms [3]. The most general form is:

$$H = \varepsilon n_d P^\dagger \cdot P + a_1 L\cdot L + a_2 Q\cdot Q + a_3 T_3\cdot T_3 + a_4 T_4\cdot T_4 \tag{1}$$

where

$$n_d = (d^\dagger \cdot \widetilde{d}),\ P = \frac{1}{2}(\widetilde{d}\cdot\widetilde{d}) - \frac{1}{2}(\widetilde{s}\cdot\widetilde{s})$$
$$L = \sqrt{10}\left[d^\dagger \times \widetilde{d}\right]$$
$$Q = \left[d^\dagger \times \widetilde{s} + s^\dagger \times \widetilde{d}\right] - \frac{1}{2}\sqrt{7}\left[d^\dagger \times \widetilde{d}\right]$$
$$T_3 = \left[d^\dagger \times \widetilde{d}\right]^{(3)}, \quad T_4 = \left[d^\dagger \times \widetilde{d}\right]^{(4)}$$

Here, n_d is the number of operator of d bosons; $s^\dagger$, $d^\dagger$ and s, d represent the s- and d- boson creation and annihilation operators, respectively.

The $E2$ and $B(E2)$ transitions

For the $E2$ transitions one uses the transition operator $T(E2)$, which is related to the quadrupole operator Q of the Hamiltonian

$$T(E2) = e_b Q = \alpha\left[d^\dagger s + s^\dagger \widetilde{d}\right]^{(2)} + \beta\left[d^\dagger \widetilde{d}\right]^{(2)} \tag{2}$$

where $\alpha(= e_b)$ and $\beta(= e_b\chi)$ are the charge parameters and are called E2SD and E2DD, respectively [4].

The $B(E2)$ branching ratio for 2 transitions from a particular level in a given band to the 2 states of other band, i.e., $(I_i \to I_f/I_f)$, depends on the Alaga value [5].The observed $B(E2)$ ratios are obtained from the γ-ray spectrum data, using the relation [6]

$$\frac{B(E2; I_i \to I_f)}{B(E2; I_i \to I_f')} = \frac{I_\gamma}{I_\gamma'} \times \frac{(E_\gamma')^5}{(E_\gamma)^5}, \tag{3}$$

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where I_γ and I'_γ are the intensities and E_γ and E'_γ are the γ-ray energies ($I_i \to I'_f$) transitions.

TABLE I: The best fit values of the Hamiltonian parameters for $^{102-108}$Ru.

Nuclei	EPS	ELL	QQ	OCT	HEX
^{102}Ru	0.698	-0.347	0.21	0.0010	–2.97504
^{104}Ru	0.621	-0.401	0.21	0.0010	–2.97504
^{106}Ru	0.604	-0.347	0.21	0.0010	–2.97504
^{108}Ru	0.548	-0.452	0.21	0.0010	–2.97504

Results and discussion

The PHINT program [7] was used to construct the IBM-1 Hamiltonian and solve it in the SU(5) basis. Input parameters EPS, PAIR, ELL, QQ, OCT, and HEX, corresponding to coefficients (EPS = ϵ, PAIR = $a_0/2$, ELL = $2a_1$, QQ = $2a_2$, OCT = $a_3/5$, HEX = $a_4/5$) (as detailed in Table I), were optimized to closely reproduce the excitation energies of all positive parity levels.

Table II presents the $B(E2)$ transition values (e^2fm^4) for the $^{98-102}$Ru nuclei, calculated using the IBM-1 framework. These values are compared with experimental data and results from the IBM-2 model. Both formalisms accurately describe intra-band transitions in the ground and γ-bands. While there are small variations in the $B(E2)$ values between the two calculations, they are generally comparable to the experimental results. In most cases, the deviations from the experimental values are less than 10%.

Conclusion

The estimated $B(E2)$ transition values are largely consistent with those from the IBM-2 model and experimental data, indicating that the IBM-1 framework effectively captures the physics of these transitions. The close alignment with experimental results reinforces the validity of both models in reflecting essential features of nuclear structure. Further investigation into the intrinsic structure and excitation mechanisms of nucleons is needed to understand the similarities between the two methods, which may involve analyzing underlying symmetries, interaction terms, and assumptions about nucleon dynamics and collective motions.

TABLE II: The best fit values of the Hamiltonian parameters for $^{102-108}$Ru.

$B(E2)e^2\mathrm{f}m^4$	Exp.	IBM-1	IBM-2
^{98}Ru			
$B(E2;2_1^+\to 0_1^+)$	78.4(24)	76.2	78.3
$B(E2;4_1^+\to 2_1^+)$	107.7(122)	105.6	108.8
$B(E2;2_1^+\to 0_2^+)$		2.6	7.1
$B(E2;2_2^+\to 0_1^+)$		1.7	0.9
$B(E2;2_2^+\to 2_1^+)$	147(25)	65.4	39.1
^{100}Ru			
$B(E2;2_1^+\to 0_1^+)$	100.2(2)	107.8	102.2
$B(E2;4_1^+\to 2_1^+)$	144.4(122)	135.7	143
$B(E2;2_1^+\to 0_2^+)$		3.4	6.5
$B(E2;2_2^+\to 0_1^+)$	4.1(46)	3.2	1.5
$B(E2;2_2^+\to 2_1^+)$	88(13)	76.4	95
^{102}Ru			
$B(E2;2_1^+\to 0_1^+)$	130.2(32)	131.3	130.1
$B(E2;4_1^+\to 2_1^+)$	211.6(233)	232.5	181.9
$B(E2;2_1^+\to 0_2^+)$		40.5	3.5
$B(E2;2_2^+\to 0_1^+)$	4.2(4)	4.4	0.6
$B(E2;2_2^+\to 2_1^+)$	117(15)	96	160.3
^{104}Ru			
$B(E2;2_1^+\to 0_1^+)$	167.0(9)	163.8	167
$B(E2;4_1^+\to 2_1^+)$	239(26)	234.9	239
$B(E2;2_1^+\to 0_2^+)$		24	6
$B(E2;2_2^+\to 0_1^+)$	0.6	3.6	5
$B(E2;2_2^+\to 2_1^+)$	167(20)	145.8	147

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