Systemic study of correlation between deformation parameters and optical parameters in quasielastic scattering

Gourab Banerjee[1] and Tinku Sinha[1*]
[1] *High Energy Nuclear & Particle Physics Division,*
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, INDIA

Introduction

The determination of ground state properties far from β-stability line is a current trend in the nuclear physics research arena. Among many other properties, the research for the study of ground state deformation of atomic nuclei has a fundamental interest not only for their roles in heavy-ion reaction dynamics, but also this microscopic interaction explores the nuclear structure properties. Recently, the quasi-elastic (QEL) scattering excitation function and its derived barrier distribution study has been adopted to determine the ground state deformation parameters in the framework of coupled channel (CC) analyses using vibrational and rotational coupling strengths. Such quadrupole (β_2) and octupole (β_3) deformation parameters could be calculated more precisely using Bayesian statistics analysis to contrast the experimental data. The quasi-elastic events consist of elastic, projectile and target excitation, and to some extent particle transfer events also. The QEL barrier distribution as a function of energy(E) is defined as the first derivative of the ratio of the QEL cross-section σ_{qel} to the Rutherford cross section σ_{Ruth}, i.e. $D_{qel}(E)=-d(d\sigma_{qel}\ /d\sigma_{Ruth})/dE$ [1]. The standard barrier distribution is evaluated using point difference formula using the step of ΔE=2 MeV in the laboratory frame. It has been proved that the behaviour of QEL barrier distribution is similar to the fusion barrier distribution from both the experimental evidence and the theoretical model. But the quasi-elastic barrier distribution has several experimental advantages over the fusion barrier distribution, such as less accuracy is required in the data taking of first derivative ($\sigma_{qel}/\sigma_{Ruth}$) rather than its second derivative. These barrier distributions provide a fingerprint of nuclear structure effects of the colliding nuclei. It is very much clear that there is a strong correlation between the relative motion of the colliding nuclei and their nuclear structure which can be observed from the systematic studies of heavy-ion reaction dynamics. The couple channel code CCFULL [2] calculates cross-sections under the influence of couplings between the relative motion of the colliding nuclei and the several nuclear collective motions. Recent work on calculation of the deformation parameters using the couple channel calculation through Bayesian optimization method is interesting [3] which finds the correlation between the different deformation parameters also. In this phenomenological work, the experimental data of QEL cross-section and its barrier distribution has been calculated for the target-projectile combination ^{90}Zr+^{16}O using CCFULL [2] and the experimental data of the literature (plots) [3] is extracted using webplot-digitizer tool [4]. Additionally, the correlation between deformation parameters β_2 & β_3 is studied using Bayesian statistics.

Analysis Methodology

In this simulation study, the experimental data has taken for a specific scattering angle in laboratory frame (θ_L) i.e. at 158° which is mentioned in the literature [3]. But the CCFULL [2] code for 'quasi-elastic scattering' takes the scattering angle as input in the center-of-mass frame. So, the input scattering angle in center-of-mass frame (θ_{cm}) is fed to the CCFULL [2]. The θ_{cm} is calculated from θ_L using numerical method following the equation (1):

$$tan\theta_L = \frac{\sin\theta_{cm}}{\cos\theta_{cm} + \frac{M_P}{M_T}} \tag{1}$$

where M_T and M_P are the masses of target nucleus and projectile nucleus respectively. The effective energy (E_{eff}) of QEL scattering cross-section and its barrier distribution is computed (CCFULL) us-

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[*] Electronic address: `tinku.sinha@saha.ac.in`

ing center of mass energy (E_{cm}) adding centrifugal correction and defined in equation (2):

$$E_{eff} = \frac{2E_{cm}\sin(\theta_{cm}/2)}{(1+\sin(\theta_{cm}/2))} \tag{2}$$

In this report, the bayesian analysis has been adopted to find the correlation between the optimized coupling strengths for the vibrational deformation parameters β_2 & β_3. This approach is based on the following Bayes theorem and given in equation (3):

$$P(\theta|D) = \frac{\mathcal{L}(D|\theta)P(\theta)}{\mathcal{Z}} \tag{3}$$

where θ and D are model parameters and experimental data respectively. Here, D is the experimental data taken from the literature [3]. $P(\theta|D)$ is the joint posterior distribution of the parameters, $\mathcal{L}(D|\theta)$ is the likelihood function, $P(\theta)$ is the prior value of the model parameters, and $\mathcal{Z}$ is the model evidence value defined as $\sum_\theta \mathcal{L}(D|\theta)P(\theta)$.

Results

In this section, we have reproduced the CCFULL [2] simulation result for quasi-elastic scattering cross-section and its barrier distribution and then compared the simulation results with the experimental data extracted using webplotdigitizer tool [4] from the plots in the literature [3]. FIG.1 shows the barrier distribution of quasi-elastic (QEL) scattering vs. effective energy (E_{eff}) only. Here, both the target and projectile have one phonon vibrational excitation. The focus of this analysis is to find the correlation or dependence between the deformation parameters and also same things for the optical parameters. We have used 100 samples in monte-carlo simulation for bayesian statistical estimation (β_2 and β_3).

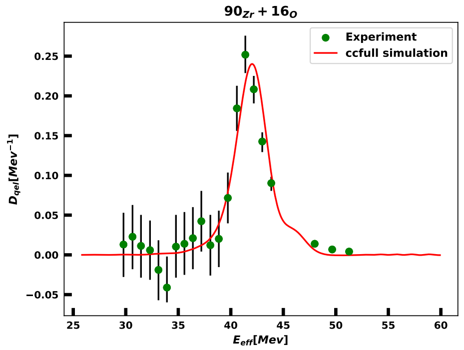

FIG. 1: The barrier distribution of quasi-elastic (QEL) scattering at laboratory angle 158° as a function of effective energy(E_{eff}).

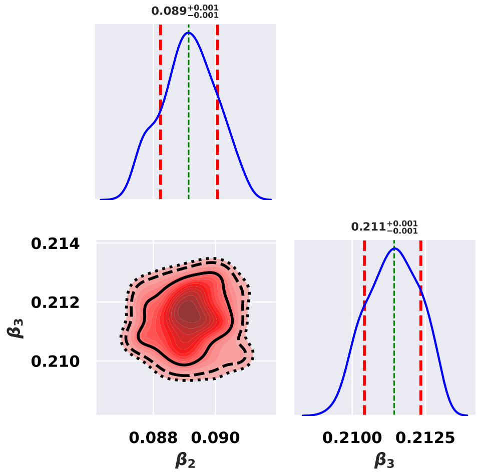

FIG. 2: The joint posterior probability distribution plot of the vibarational deformation parameters (β_2 & β_3).

Summary

In this analysis, the very minimal positive correlation (5%) between these two parameters (β_2 and β_3) has been found as shown in FIG.2. We are working on a systematic study where the same dependencies are analyzing from lower massive system to higher massive system. The work is in progress.

References

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