A Comparative Study of Kernels in Nonlocal Nuclear Interactions

N. J. Upadhyay[1*] and A. Bhagwat[2†]

[11]*Department of Physics, Amity School of Applied Sciences, Amity University Maharashtra, Panvel - 410206, INDIA and*

[2]*School of Physical Sciences, UM-DAE Centre for Excellence in Basic Sciences, University of Mumbai, Vidyanagari Campus, Mumbai-400098, INDIA*

Introduction

The nuclear potential is known to exhibit nonlocal characteristics [1] that necessitates the use of integro-differential equation, written as:

$$\left[\frac{\hbar^2}{2\mu}\nabla^2 + E\right]\Psi(\mathbf{r}) = \int K(\mathbf{r},\mathbf{r}')\Psi(\mathbf{r}')d\mathbf{r}' \,, \tag{1}$$

where $K(\mathbf{r},\mathbf{r}')$ is the nonlocal kernel, μ is the reduced mass of the system, E is the center of mass energy and $\Psi(\mathbf{r})$ is the scattering wave function. of the nonlocal interaction. The partial wave expansion of Eq.(1) results in the radial equation written as

$$\hat{\mathcal{L}}\, u_{jl}(r) \;=\; \frac{2\mu}{\hbar^2}\int_0^\infty g_l(r,r')u_{jl}(r')dr', \tag{2}$$

where, $\hat{\mathcal{L}} \equiv \left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2\mu\, E}{\hbar^2}\right]$,

$$g_l(r,r') = \; 2\pi r r' \int_{-1}^{1} K\left(\mathbf{r},\mathbf{r}'\right)\; P_l\left(\cos\theta\right) d\left(\cos\theta\right) .$$

In the present work, a separable form for the nonlocal kernel is used, which is given as

$$K\left(\mathbf{r},\mathbf{r}'\right) \;=\; H(|\mathbf{r}-\mathbf{r}'|)\, U\left(\frac{|\mathbf{r}+\mathbf{r}'|}{2}\right). \tag{3}$$

The Gaussian form of $H(|\mathbf{r}-\mathbf{r}'|)$ [2], written as

$$H(|\mathbf{r}-\mathbf{r}'|) = \frac{1}{\pi^{3/2}\beta^3}\exp\left[-\frac{|\mathbf{r}-\mathbf{r}'|^2}{\beta^2}\right], \tag{4}$$

is the popular choice to describe the scattering cross-sections due to their simplicity. However, it may not be able to accurately capture the nuanced features of nuclear interactions, particularly those involving strong repulsive cores or long-range tails. Further, the complexity of nuclear interaction is oversimplified, which is often influenced by factors such as exchange effects, meson exchange and relativistic effects [3].

In this study, we address the above limitations by exploring other functional forms of $H(|\mathbf{r}-\mathbf{r}'|)$ that can lead to more accurate and physically meaningful results.

Other Functional Form of $K\left(\mathbf{r},\mathbf{r}'\right)$

As an example, we consider a Trident-shaped kernel that may allow for a better description of nuclear interactions involving both repulsive and attractive components. The mathematical form for a Trident-shaped kernel is:

$$H(|\mathbf{r}-\mathbf{r}'|) = \frac{8}{17\pi^{3/2}\beta^3}\left\{\exp\left[-\frac{|2\mathbf{r}-\mathbf{r}'|^{\mathbf{2}}}{\beta^2}\right] + \exp\left[-\frac{|\mathbf{r}-\mathbf{2r}'|^{\mathbf{2}}}{\beta^2}\right] + \exp\left[-\frac{|\mathbf{r}-\mathbf{r}'|^{\mathbf{2}}}{\beta^2}\right]\right\} \tag{5}$$

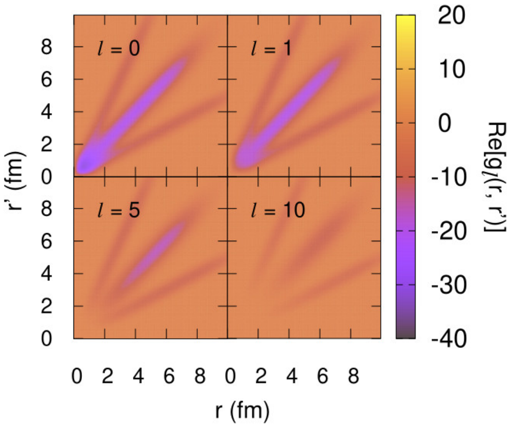
The β is the nonlocal range parameter whose value is taken to be 0.9 fm. In this work, the $U\left(\frac{|\mathbf{r}+\mathbf{r}'|}{2}\right)$ is taken to be of Wood-Saxon form with parameters taken from Ref.[4]. This potential will be referred as “TPM15”.

Results and Discussions

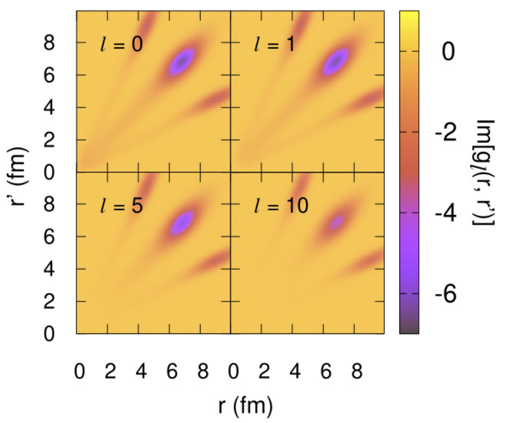
As an illustration, in Fig.(1) we show the Trident-shaped nonlocal kernel for neutron scattering off ^{208}Pb using the TPM15 potential. As can be seen from the figure, the kernel

*Electronic address: `nupadhyay@mum.amity.edu`

†Electronic address: `ameeya@cbs.ac.in`



(a)Real part



(b)Imaginary part

FIG. 1: Behavior of (a) real part and (b) imaginary part of Trident-shaped kernel as a function of distance for different l. Calculations are done for neutron scattering off ^{208}Pb using the TPM15 potential.

is dominated by a central peak surrounded by two smaller peaks.

Using the Trident-shaped kernel, the integro-differential equation in Eq.2 is solved using Iterative Perturbation Approach (IPA) developed in Ref.[2]. With the motivation to study convergence properties of IPA for such kernels, in Fig.(2), we show the total cross sections for neutron scattering off ^{208}Pb at various energies. For calculations, 11 partial waves are considered. As can be observed, up to 10 MeV we obtain convergence within 50 iterations. However, beyond 10 MeV, same degree of convergence is achieved with possibly a larger number of iterations. Computationally, change in the shape of the kernel increases the run time when compared with that required for the Gaussian kernel.

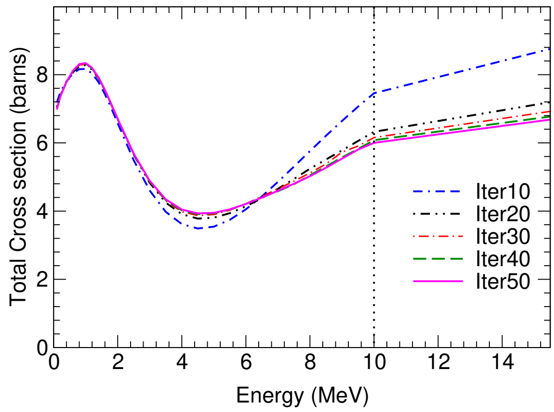


FIG. 2: Convergence properties of the IPA method. Total cross sections for neutron scattering off ^{208}Pb as a function of incident energy for different iterations.

The convergence obtained here demonstrates that the IPA method works even for the kernels that do not peak around $r = r'$, thereby making the method applicable to realistic kernels obtained from the many-body calculations. Study along these lines is in progress.

References

[1] H. A. Bethe, Ann. Phys.(N.Y.)**3**, 190 (1958).

[2] N. J. Upadhyay & A. Bhagwat, Phys. Rev. C**98**, 024605 (2018).

[3] J. W. Negele and D. Vautherin, Phys. Rev C**5**, 1472 (1972).

[4] Y. Tian, D.-Y. Pang, and Z.-Y. Ma, Int. J. Mod. Phys. E **24**, 1550006 (2015).