Estimating Scattering Potentials for Nucleon-Nucleon Scattering using Physics Informed Machine Learning

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Introduction

The nucleon-nucleon (NN) interaction remains an open problem in nuclear physics. Groups like Argonne, Bonn, and Nijmegen have developed high-precision NN potentials using one-pion exchange at long range (r $\geq$ 2.0 fm) and various interaction terms like central, tensor, spin-spin, spin-orbit, tensor etc. at intermediate (1.0 < r < 2.0) fm and shorter ranges (r < 1.0 fm) to fit phase shifts [1]. Alternatively, the phase function method is way estimates potentials directly from scattering observables such as phase shifts. This is conceptually similar to machine learning. In this work, we apply a Physics-Informed Machine Learning (PIML) approach to estimate the inverse potential for the $^1S_0$ NN scattering state by minimizing the mean square error (MSE) between simulated and observed phase shifts using a piecewise Morse function as a reference model [2].

Methodology

We employ a PIML framework, integrating the phase function method to estimate the $^1S_0$ NN scattering potential [2].

A. Phase Function Method

The Schrödinger equation for $\ell = 0$ is transformed into a Ricatti-type equation:

$$\delta_0'(k,r) = -\frac{V(r)}{(\hbar^2/2\mu)k}\sin^2[kr + \delta_0(r)], \qquad (1)$$

where $k_{c.m} = \sqrt{E_{c.m}/(\hbar^2/2\mu)}$. This non-linear equation is solved using the RK-5 method with initial condition $\delta_0(k,0) = 0$. Using phase shifts as input, the potential is estimated with a piecewise Morse function as the reference model [2].

B. Optimization Using Machine Learning

We optimize the model parameters of the reference model [2] using machine learning based algorithm such as genetic algorithm (GA) [3]. GA is versatile, suitable for optimizing discontinuous, non-differentiable, stochastic, or highly non-linear functions. Moreover, GAs are easily parallelizable, fast, and capable of exploring vast search spaces efficiently. They can accommodate multiple complex optimization objectives. Using this algorithm, we optimized the model parameters by minimizing the loss function called MSE, defined as

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(\delta^i_{inp}(kr) - \delta^i_{obt}(kr))^2,$$

Here, $\delta^i_{inp}(kr)$ represents the input phase shifts from the Wiringa *et. al.* [4], while $\delta^i_{obt}(kr)$ are the optimized values obtained by solving the phase equations. The input phase shifts, defined at various energies and angular momenta ℓ, are used to infer the unknown potential $V(r)$. By optimizing the model parameters with GA, we reconstructed the inverse potentials. The optimization is guided by the phase function method to ensure physical consistency. The Physics-Informed Machine Learning approach integrates physics into the process, effectively identifying the best-fit parameters for the inverse scattering potential.

Results and Discussion

The scattering phase shift data up to 350 MeV for the $^1S_0$ state of neutron-proton, proton-proton, and neutron-neutron interactions, provided by Wiringa *et al.* [4], is used as input to

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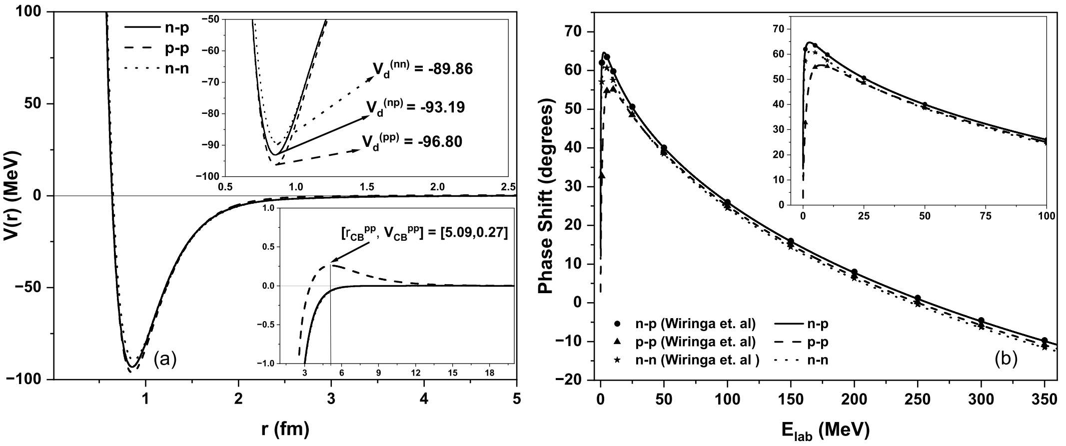

FIG. 1: Scattering Potential for $^1S_0$ state of n-p, p-p and n-n scattering along with their corresponding scattering phase shifts.

the phase equation to estimate the scattering potentials. To compute the inverse potentials for the $^1S_0$ state, we explore a 10-dimensional parameter space. Optimization is performed using a machine learning-based algorithm, where the choice of parameter bounds plays a critical role. Initially, the bounds were set as: $[\alpha_0, \alpha_1, \alpha_2, r_0, r_1, r_2, V_2, x_1, x_2, D_0]$ = [(0.01, 10), (0.01, 10), (0.01, 10), (0.01, 6), (0.01, 10), (0.01, 10), (0.01, 5), (0.01, 1), (1.01, 4), (0.01, 500)], resulting in a large sample space. After analyzing the optimized parameters from several thousand iterations, the bounds were refined to [(0.01, 2), (0.01, 10), (0.01, 2), (0.01, 6), (0.01, 2), (0.01, 5), (0, 0.01), (0.01, 1), (1, 4), (0.01, 100)] to reduce computational time. The mean square error (MSE) of the best solution, representing the potential accurately, was on the order of 10^{-3}. With the optimized parameters, we constructed the inverse potentials and calculated the corresponding scattering phase shifts by solving the phase equation, as shown in Fig.1. From this figure, we observe that the potential depths for n-p, p-p, and n-n interaction are -93.19 MeV, -96.80 MeV, and -89.86 MeV, respectively. The small difference of 4-5 MeV between these values is expected due to the differences in phase shifts. However, the overall nature of the interactions remains consistent across all three cases, indicating the charge independence of nuclear forces. From Fig.1(b), we observed that phase shifts exhibit a decreasing trend, remaining positive from 1 MeV to 250 MeV, but turning negative between 300 and 350 MeV. This behavior indicates that the constructed inverse potential is attractive up to 250 MeV, with strong repulsion occurring at shorter inter-nuclear distances at higher energies. The inverse potentials obtained for $^1S_0$ state are phenomenological, as they account for the full range of interactions between the scattering particles. The strong agreement between our potentials and existing high-precision realistic potentials, which consider internal interactions in detail, confirms the validity of our computational method. This opens the door to an alternative approach for exploring the fundamental nature of interactions in various scattering processes.

Acknowledgments

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