Bayesian Inference for Modeling Pulsar Glitch Recovery

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Introduction

Neutron Stars are highly magnetized, extremely dense, rapidly rotating remnants of a massive star. Radiations emitted by some of this class of stars appear as pulses to a distant observer, hence they are known as pulsars. Pulsars generally show remarkable uniformity in their rotation rates. However, occasionally, the pulsar's frequency undergoes a sudden increase, known as a 'pulsar glitch' [1]. Glitch is an intriguing rare phenomenon and occurs due to variations of coupling of superfluid in the neutron star's interior with its surface. These glitches may cause permanent changes in the spin frequency and its derivatives. After a glitch, the pulsar follows a relaxation phase. The recovery from a glitch may be slow and may not be complete until the next one occurs.

In this work, we model variation of the change in the frequency ($\Delta\nu$) during the post-glitch recovery phase of the Vela glitch at MJD 57734. We also report the values of estimated parameters which can then be used to determine the fractional changes in the moment of inertia and to constrain the neutron star equation of states.

Formalism

To understand the post-glitch behaviour, we utilised various models from the literature [2, 3, 4]. We have categorized these models into five categories as follows:

(i) Model I

$$\Delta\nu = \Delta\nu_{\rm p} + e^{\frac{-(t-t_{\rm g})}{\tau_{\rm d}}}\Delta\nu_{\rm d} \qquad (1)$$

(ii) Model II

$$\Delta\nu = \Delta\nu_{\rm p} + \Delta\dot{\nu}_{\rm p}(t-t_{\rm g}) \qquad (2)$$

(iii) Model III

$$\Delta\nu = \Delta\nu_{\rm p} + \Delta\dot{\nu}_{\rm p}(t-t_{\rm g}) + e^{\frac{-(t-t_{\rm g})}{\tau_{\rm d}}}\Delta\nu_d \qquad (3)$$

(iv) Model IV

$$\Delta\nu = \Delta\nu_{\rm p} + \Delta\dot{\nu}_{\rm p}(t-t_{\rm g}) + \frac{1}{2}\Delta\ddot{\nu}_{\rm p}(t-t_{\rm g})^2 \qquad (4)$$

(v) Model V

$$\Delta\nu = \Delta\nu_{\rm p} + \Delta\dot{\nu}_{\rm p}(t-t_{\rm g}) + \frac{1}{2}\Delta\ddot{\nu}_{\rm p}(t-t_{\rm g})^2 + \left(1 - e^{\frac{-(t-t_{\rm g})}{\tau_{\rm d}}}\right)\Delta\nu_d\tau_d \qquad (5)$$

where $\Delta\nu_{\rm p}$ is the permanent change in the frequency, $\Delta\dot{\nu}_{\rm p}$ is the permanent change in the first time derivative of the frequency, $\Delta\ddot{\nu}_{\rm p}$ is the permanent change in the frequency second time derivative induced by a glitch, and $t_{\rm g}$ is the glitch epoch. $\Delta\nu_d$ and τ_d are the amplitude and the relaxation time scale of the decaying component, respectively, expressing the exponential recovery phase. These sets of models are essential as the post-glitch recoveries show various features [2, 4, 5].

For model selection and parameter estimation, we utilised Bayesian statistics based on Bayes' Theorem. To select the most preferred model, we estimated the Bayes factor, which is defined as the ratio of evidence. Jeffrey's scale is utilised to examine the preference of one hypothesis or model over another. The parameters for the most preferred models were calculated using the Bayesian parameter estimation approach.

Results and Discussions

A glitch in the Vela pulsar was reported to occur at MJD 57734.4(2) with a fractional amplitude $\Delta\nu/\nu$ of 1433(3) $\times$ 10^{-9} [3]. We performed Bayesian Model selection on the post-glitch recovery observations, which suggested

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that Model IV is the most preferred model among the five models described in the previous section.

The post-glitch relaxation modeling for the glitch in the framework of Model IV is shown in Fig. 1. We utilised Bayesian parameter estimation to calculate the permanent changes in spin frequency and its derivatives. The estimated parameters for the most preferred model are given in Table I, and the corresponding corner plots obtained using the Bayesian analysis are presented in Fig. 2.

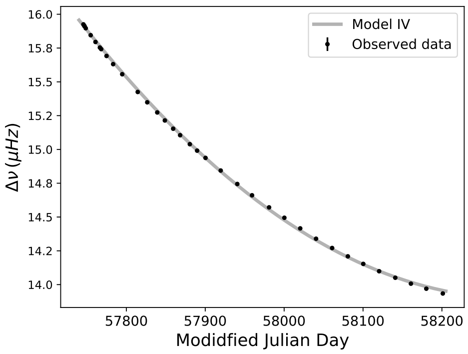

FIG. 1: The post-glitch recovery modeling within the framework of Model-IV.

Parameter	Value
$\Delta\nu_{\rm p}\,(\rm Hz)$	$1.59690^{+0.00001}_{-0.00001} \times 10^{-5}$
$\Delta\dot{\nu}_{\rm p}\,(\rm Hz\,s^{-1})$	$-8.6336^{+0.0008}_{-0.0008} \times 10^{-14}$
$\Delta\ddot{\nu}_{\rm p}\,(\rm Hz\,s^{-2})$	$1.8011^{+0.0003}_{-0.0003} \times 10^{-21}$

TABLE I: The calculated parameters using Model IV with Bayesian parameter estimation.

The future work includes modeling the post-glitch recovery for various glitches detected in our monitoring program [3, 5] to report the permanent changes in the rotation of the pulsar. We intend to utilise those results to find correlations and to put observational constraints on the neutron star equation of state.

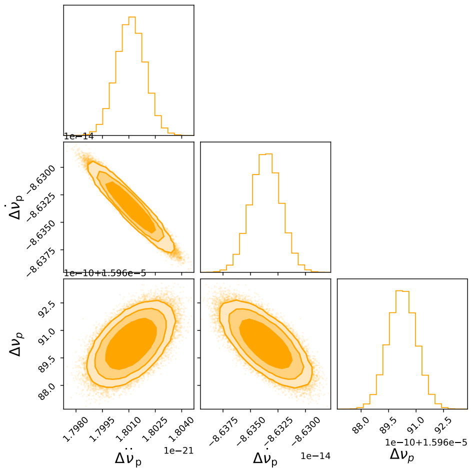

FIG. 2: The posterior diagram for the recovery modeling. The contours are plotted with 68, 95 and 99% credible intervals.

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