Study of $p-^{12}$C at Astrophysical Energies Using Phase Function Method

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Introduction

The p-^{12}C reaction is crucial for energy production in stars, particularly through the CNO cycle [1]. Studying this reaction in the astrophysical energy range (0–10 keV) provides key insights into stellar processes. In this work, we model the interaction potential for proton energies between 400 keV and 1000 keV in the elastic region, focusing on scattering from the $^2S_{1/2}$ ($\ell = 0$) resonant state, which predominantly contributes to the astrophysical S-factor. The inverse potentials are constructed using the phase function method (PFM), which directly obtains phase shifts from the potential without relying on the wave function. The nuclear interaction between the proton and ^{12}C is modeled using the Morse potential, while the screened Coulomb interaction is described by the atomic Hulthén potential.

Methodology

A. Modeling the Interactions

The nuclear part of the $p-^{12}$C interaction, is modeled by Morse potential and the long-range Coulomb interaction, which gets typically screened in experimental scenario, is taken to satisfy the Atomic Hulthén form of the potential [2]. hence, the net model of interaction is given by

$$V(r) = V_0\left(e^{\frac{-2(r-r_m)}{a_m}} - 2e^{\frac{-(r-r_m)}{a_m}}\right) + V\frac{e^{-r/a}}{1-e^{-r/a}}$$

where V_0 represents the nuclear potential strength, a_m is the shape parameter, r_m is the equilibrium distance, V is the strength of the Coulomb barrier and a is the screening parameter. V and a are related to the Sommerfeld parameter η as:

$$aV = 2k\eta = \frac{2Z_1Z_2e^2\mu c^2}{\hbar^2c^2} = 0.384 \text{ fm}^{-1}$$

B. Phase Function Method (PFM)

The second order time-independent Schrödinger equation for $\ell = 0$ is transformed into nonlinear Riccati equation, given as:

$$\delta_0'(k,r) = -\frac{U(r)}{k}\sin^2[kr + \delta_0(r)]$$

where $k = \sqrt{E_{c.m}/(\hbar^2/2\mu)}$. The Runge-Kutta method is employed to solve this equation using the initial condition $\delta_0(k,0) = 0$.
Mean Absolute Percentage Error (MAPE) is used as the cost function in the optimization procedure driven by Genetic algorithm [3], which is given as:

$$\text{MAPE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\delta^i_{inp} - \delta^i_{sim}}{\delta^i_{inp}}\right| \times 100$$

where δ^i_{inp} represents the expected phase shift values[1], and δ^i_{sim} represents the calculated values from our simulation procedure.

Result and Discussion

The available scattering phase shift data for the S-state of $p-^{12}$ C ranges a little below 400 keV to 1 MeV. There are only six data points that capture the sharp variation in phase shifts from 400 to 500 keV and the rest of the 25 data points which are closely spaced between 500 to 1000 keV fall almost on a horizontal line with an extremely small negative slope. Trying to consider all the points in the data during optimization leads to poor convergence in the rising part of the phase

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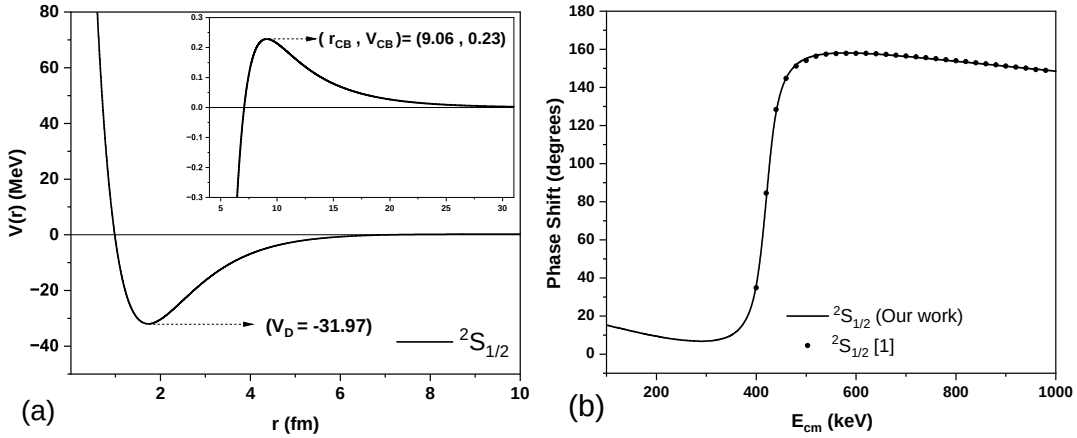

FIG. 1: (a) Interaction potential as a function of r, (b) Phase shifts plotted as a function of proton energy in the center-of-mass frame.

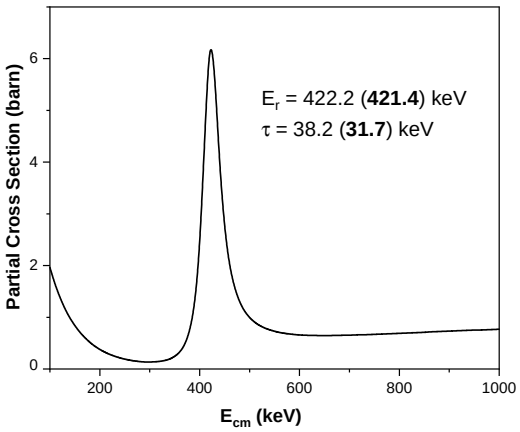

FIG. 2: Partial cross-section as a function of E_{cm}. Experimental values are given in [1] and are shown in parentheses.

shift curve. Hence, we have chosen only a few data points for higher energies, while minimizing the mean absolute percentage error. Hence, the four-dimensional parameter space, that covers all possible representative potential curves, is initialized with random values within defined bounds, and then they are updated in an iterative fashion by using a genetic algorithm [3], till the mean absolute percentage error converges to a minimum value. The optimized model parameters for the $p - {}^{12}$C interactions, as summarized in Table I. The resultant potential is plotted in FIG.1(a) and

TABLE I: Optimized model parameters and the *MAPE* value for $p - {}^{12}$C system:

$\mathbf{V_0(MeV)}$	$\mathbf{r}_m\mathbf{(fm)}$	$a_m\mathbf{(fm)}$	$\mathbf{a(fm)}$	**MAPE**
36.17	1.68	1.09	4.72	0.3

the corresponding scattering phase shifts obtained are shown in FIG. 1(b). From FIG. 1(a), potential depth is found to be V_d = -39.17 MeV and the Coulomb barrier V_{CB} is found to be 0.23 MeV at r_{CB} = 9.06 fm. As illustrated in FIG. 1(b), the simulated SPS closely align with the expected phase shifts [1], indicating the accuracy of our model.

Utilizing the obtained scattering phase shifts, we determined the partial scattering cross sections at different energies and the plot is presented in FIG.2. From this plot, the resonance energy is obtained(**experimental**) to be E_r = 422.2 **(421.4)** keV and resonance width τ = 38.2 **(31.7)** keV. These findings highlight the effectiveness of our approach in accurately characterizing nuclear interaction dynamics.

References

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