Simulation of Binary Neutron Star Merger Dynamics and r-Process Nucleosynthesis

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Introduction

Neutron stars are highly densed astrophysical objects which exit sometimes in orbiting pair known as Binary Neutron Star (BNS). The study of BNS mergers has gained considerable attention in recent years, particularly after the groundbreaking observation of gravitational waves from such an event by the LIGO and Virgo collaborations in 2017. In this work we have studied BNS merger trajectory with and without post-Newtonian correction and also have calculated tentative mass ejecta by predicting an emperical relation focussing on r-process nucleosynthesis.

Method

The neutron stars of masses 2.09 (m_1) and 2.16 (m_2) solar mass are placed at an initial separation of 10^7 meters along the x-axis. Iterative update of positions and velocities and ploting the trajectory are simulated using python codes for the numerical methods employed (e.g. $solve_{ivp}$) to solve the relevant differential equations. Then we have employed post-Newtonian corrections in the Peters-Mathews formula (Equation (16)[1]) retaining first term only considering circular orbit to find orbital loss of energy due to emission of gravitational wave as:

$$\frac{dE}{dt} = -\frac{32}{5}\left(\frac{G^4\mu^2M^3}{c^5r^5}\right) \tag{1}$$

Relevant derivatives are computed as,
Radial acceleration:

$$\frac{dv_r}{dt} = rv_\theta^2 - \frac{G(m_1+m_2)}{r^2} - \frac{2v_rE}{\mu c^2} \tag{2}$$

1st term is centrifugal acceleration, 2nd is gravitational attraction between two stars in binary, and the final term is the contribution from the loss of energy due to gravitational radiation. v_r and v_θ are radial and cross-radial velocities respectively.
Anguular acceleration:

$$\frac{dv_\theta}{dt} = -\frac{2v_rv_\theta}{r} \tag{3}$$

Where, $M = m_1 + m_2$ and $\frac{1}{\mu} = \frac{m_1+m_2}{m_1m_2}$. Initial orbital velocity of BNS is $v = \sqrt{\frac{G(m_1+m_2)}{r}}$ Numerical solutions of differential equations are done using $solve_{ivp}$ and the trajectory of the neutron stars is plotted. We have used an empirical relation to estimate the mass of the ejecta based on the masses of the neutron stars as follows:

$$M_{ej} = 0.04\left(1+\frac{m_1}{m_2}\right)^{-1.5}\left(\frac{m_1m_2}{M^2}\right)^{0.5} \tag{4}$$

Starting with an initial abundance of isotope A, a system of differential equations are employed to simulate the conversion rates between isotopes A, B, and C. The rates of reaction from A to B and B to C are taken as 0.1 and 0.05 respectively. We have used odeint to solve the differential equations for the isotope abundances over time and relevant abundances over time are plotted.

Results and Observations

Fig 1 shows that the neutron stars follow spiral trajectories as they move towards each other. The spiral paths become tighter as the simulation progresses, indicating a decreasing separation between the neutron stars. The Fig 2 shows the trajectory of the binary neutron stars. Compared to the previous plot, this one accounts for post-Newtonian corrections, providing a more accurate representation of the orbital decay. The Fig 3 shows the evolution of isotope abundances over time: Isotope A decreases rapidly initially, indicating a quick conversion to isotope B. Isotope B peaks around 10-20 units of time and then decreases as it converts to isotope C which increases steadily, eventually becoming the most abundant isotope. The estimated mass of the ejecta is approximately 6.90×10^{-3} solar masses.

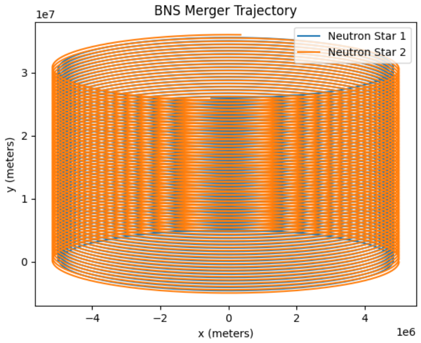

Figure 1: BNS Merger Trajectory

Conclusions

The inclusion of post-Newtonian corrections provides a more accurate and realistic simulation of the binary neutron star merger trajectory. The mass ejecta estimation aligns well with experimental observation [2]. The r-process nucleosynthesis simulation effectively models the conversion of isotopes, highlighting the production of heavy elements in the aftermath of the merger.

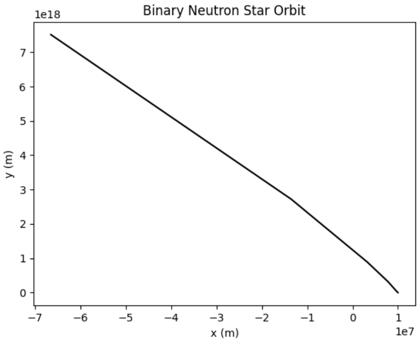

Figure 2: BNS Merger Trajectory With Post-Newtonian Correction

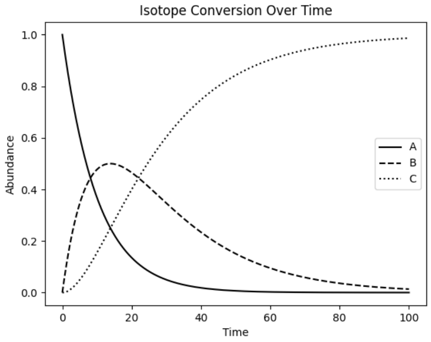

Figure 3: R-Process Nucleosynthesis Isotopes Abundances

Overall, the enhanced simulation offers deeper insights into the dynamics of neutron star mergers and the associated nucleosynthesis processes, contributing valuable information to the field of nuclear astrophysics.

References

1. Peters P.C. and Mathews J (1963), Gravitational Radiation from Point Masses in a Keplerian Orbit, Physical Review, 131, 435-440.
2. Dietrich, T., et al. (2017), Modeling dynamical ejecta from binary neutron star mergers, Class. Quantum Grav. 34 (2017) 105014 (25pp)