Sensitivity of radiative capture cross sections to the bound state wave functions

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Introduction

In the area of nuclear astrophysics, radiative capture reactions [denoted as $b + c \rightarrow a + \gamma$ or $b(c, \gamma)a$, where c is the particle being captured by nucleus b] are of great interest as they play an important role in the synthesis of nuclei in stars and in other astrophysical scenario. Most common cases involve radiative capture of a nucleon (proton or neutron) or an α particle. The capture process can be a compound nuclear reaction or a direct process, but in the light nuclei it is the direct capture mechanism which dominates in most of the cases.

Theoretically, there exist various models to study the radiative capture reactions which are based upon the direct as well as indirect approaches such as Coulomb dissociation [1]. Among them, the potential model is a simpler and quite popular model to study the direct capture reactions with the assumption of two structureless particles interacting via a potential that depends upon the relative coordinates. One needs to construct bound and countinuum states using suitable potentials. In most of the models, two-body potentials such as Woods-Saxon (WS) or Gaussian type are normally adopted where the bound states are constructed by adjusting the potential parameters to reproduce the experimental binding energies. However, these two-body potentials constructed for nuclear many-body systems contain Pauli-forbidden states, which are unphysical but simulate the missing antisymmetrisation effects. These deep unphysical states, on the other hand, can be removed by invoking supersymmetric quantum mechanics which can transfer a potential to its supersymmetric (SUSY) partner [2]. These SUSY potentials are shallow, $\ell-$dependent and are characterised by a repulsive core. Although, at positive energies they give the same phase shift but the corresponding wave functions differ in the nuclear interior where they posses a node-less structure. It has been found that inspite of their different behaviour in the nuclear interior, both the wave functions give similar spectroscopic properties. We intend to study the effect of SUSY potentials on the cross sections of key nuclear reactions important for astrophysics.

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Formalism

The direct radiative capture cross section $\sigma^{cap}_{\pi\lambda}$ for the process $b + c \rightarrow a + \gamma$, is proportional to the square of its reduced transition probability

$$\sigma^{cap}_{\pi\lambda} \propto |\langle\psi_i||O_{\pi\lambda}||\psi_f\rangle|^2, \qquad (1)$$

where $\pi\lambda$ stands for electric or magnetic transition of multipolarity λ. $O_{\pi\lambda}$ is the multipole operator for the electric/magnetic transitions and is responsible for changing the initial continuum state (for the relative motion of b and c) to a final bound state (of nucleus a) while simultaneously creating a photon of proper character, multipolarity and energy. The initial and final state wave functions (ψ_i and ψ_f) are contained in the reduced transition matrix element which are calculated using the potential model [3]. For a complete expression of $\sigma^{cap}_{\pi\lambda}$ and for more details one is referred to Ref. [3–5].

As mentioned earlier, the radial wave functions can be obtained by solving the Schrödinger equation with the WS potential, where, for the bound state we adjust the potential parameters in order to reproduce the particle separation energy. In case the initial state is a resonance, one can adjust the potential parameters so as to reproduce the resonance energy but on the other hand, widths might not be precisely reproduced by the model. Then, the cross sections can be computed for a particular transition of a specific multipolarity. To study the sensitivity of cross sections to the bounds state wave functions we then replace the final bound state potential by its SUSY partner potential. We are mainly concerned with the cases where final bound state has one or more nodes which can be removed in the new wave function obtained from the SUSY potential. In fact, in supersymmetric quantum mechanics a Hamiltonian can be transformed to its partner Hamiltonian having the identical bound-state energy spectrum except for the lowest state which get suppressed by performing couple of transformations [2].

Results and discussions

We are interested in studying several cases of nucleon and alpha capture reactions using the procedure mentioned above. As a first case, we consider the $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ reaction which is an important astrophysical reaction. It is the primary source of energy production in more massive main sequence stars and being the slowest process in the hydrogen burning CNO cycle, it governs the energy production of the cycle.

The major contribution to its non-resonant cross sections comes from the $E1$ transitions from the continuum to the 6.793 MeV $(3/2^+)$ excited state in ^{15}O, which can be obtained by coupling the $2s_{1/2}$ proton to ^{14}N (1^+) core. With one Pauli-forbidden state in the final potential, this is a perfect case for our study.

In Fig. 1, we plot the WS bound state potential (solid line) for the $3/2^+$ state of ^{15}O, for which the potential parameters are adopted from Ref. [6]. For a comparison, we also plot its SUSY partner (dashed line) which is shallow but has a repulsive core. Corresponding wave functions will be used to calculate the radiative capture cross section to study the influence of nuclear interior on the capture cross sections.

We shall present results of our calculations for several key nuclear reactions having importance in astrophysics including cases of nucleon as-well-as alpha capture.

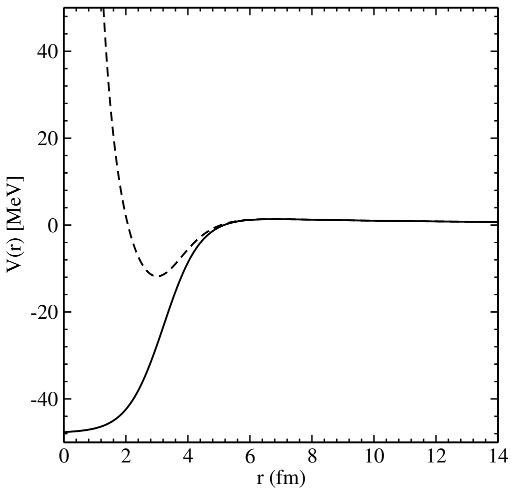

FIG. 1: Woods-Saxon potential (solid line) for the ^{15}O $(3/2^+$, 6.793 MeV) excited state. For a comparison, we also plot its SUSY partner (dashed line) which is shallow but has a repulsive core.

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