Universal relations for neutron stars

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Introduction

Utilizing a perturbative method and canonical (APR) and Brussels-Montreal Skyrme (BSk22, BSk24, BSk26) equations of state (EoSs) characterizing the hadronic matter of neutron stars, the universal relationships for compact stars have been studied. The β-equilibrated neutron-proton-electron-muon matter at the core with a rigid crust, has been proposed as the composition of neutron star (NS) matter. A NS that rotates slowly has an exterior gravitational field that is characterized by its multipole moments. These factors depend on the internal structure of NS determined by the EoS. The mass, radius, dimensionless moment of inertia, Love number, dimensionless tidal deformability and dimensionless quadrupole moment of NSs have all been estimated and relationships between these quantities have been investigated.

Theoretical formalism

The quadrupole moment Q_{ij} of compact star and the external tidal field $\mathcal{E}_{ij}$ are defined as coefficients in an asymptotic expansion of the total metric at large distances r from the star. The tidal Love number k_2 depends upon EoS and characterizes the response of NS to the tidal field $\mathcal{E}_{ij}$ [1]. This relation can be expressed as

$$Q_{ij} = -\frac{2}{3}k_2 R^5 \mathcal{E}_{ij} = -\lambda \mathcal{E}_{ij} \qquad (1)$$

where R is radius and $\lambda = 2k_2R^5/3$ is tidal deformability of NS. The tidal Love number for $l = 2$ is given by [2, 3]

$$\begin{aligned} k_2 &= \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \\ &\times\Big\{2C[4(1+y)C^4+(6y-4)C^3+(26-22y)C^2 \\ &+3(5y-8)C-3y+6]+3(1-2C)^2 \\ &\times[2C(y-1)-y+2]\ln(1-2C)\Big\}^{-1} \end{aligned} \qquad (2)$$

where we use units $G = c = 1$ throughout and the dimensionless quantity $C = M(R)/R$ is the compactness of the star and the dimensionless quantity $y = \frac{R\,\beta(R)}{H(R)}$ is defined as for the internal solution determined by numerically solving the second-order differential equation for $H(r)$. The second-order differential equation for H can be separated into a first-order system of ordinary differential equations in terms of the usual TOV quantities mass $M(r)$, pressure $p(r)$ and energy density $\varepsilon(p)$ as well as the additional functions $H(r)$, $\beta(r) = dH/dr$ and the EoS function $f(p) = d\varepsilon/dp$:

$$\frac{dH}{dr} = \beta \qquad (3)$$

$$\begin{aligned} \frac{d\beta}{dr} &= 2\left(1-2\frac{M}{r}\right)^{-1} H\Big\{-2\pi\left[5\varepsilon+9p+f(\varepsilon+p)\right] \\ &+\frac{3}{r^2}+2\left(1-2\frac{M}{r}\right)^{-1}\left(\frac{M}{r^2}+4\pi r p\right)^2\Big\} \\ &+\frac{2\beta}{r}\left(1-2\frac{M}{r}\right)^{-1}\Big\{-1+\frac{M}{r}+2\pi r^2(\varepsilon-p)\Big\} \end{aligned} \qquad (4)$$

The above equations combined with TOV equations is then solved simultaneously. The system is integrated outward starting just outside the center using the expansions $H(r) = a_0r^2$ and $\beta(r) = 2a_0r$ as $r \to 0$. The constant a_0 determines how much the star is deformed and can be chosen arbitrarily as it cancels in the expression for the Love number. The boundary conditions that determine the unique choice of this solution follow from matching the interior and exterior solutions and their first derivatives at the boundary of the star, where $r = R$. The dimensionless tidal deformability related to the Love number by $\Lambda = \frac{2}{3}k_2C^{-5}$. One of the primary quantities that can be accurately determined by detecting associated gravitational waves (GWs) is the tidal deformability or Λ. By introducing the function $\varpi = \Omega - \omega$, the angular velocity of a point in the star measured with respect to the angular velocity ω of the local inertial frame, it can be shown that it satisfies the following equation [1, 3]

$$\frac{1}{r^4}\frac{d}{dr}(r^4 j\frac{d\varpi}{dr}) + \frac{4}{r}\frac{dj}{dr}\varpi = 0, \qquad (5)$$

where $j(r) = e^{-\nu/2}\sqrt{1-\frac{2M(r)}{r}}$ which $\to 1$ at $r \to R$. The Eq.(5) can be solved subject to the boundary conditions that $\varpi(r)$ is regular as $r \to 0$ and $\varpi(r) \to \Omega$ as $r \to \infty$. The moment of inertia of the star can then be calculated by dividing the total angular momentum J by angular velocity Ω of the star as

$$\mathcal{I} = \frac{1}{6\Omega}R^4\frac{d\varpi}{dr}\Big|_{r=R}. \qquad (6)$$

In the dimensionless form moment of inertia I of the star is given by $I = \mathcal{I}/M^3$. The quadrupole moment $\mathcal{Q}$ can be determined in terms of K as $\mathcal{Q} = \frac{J^2}{M} + \frac{8}{5}KM^3$ which can be expressed in the dimensionless form as $Q = \frac{\mathcal{Q}}{M^3(J/M^2)^2} = 1 + \frac{8}{5}K\frac{M^4}{J^2}$ where K is determined from continuity condition at $r = R$ while solving Eq.(5).

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Calculations and Results

Universal relations among I, Λ and Q have been explored using APR, BSk22, BSk24 and BSk26 EoSs. It is observed that these relations hold universally for NS sequences, essentially independently of their EoSs. Such relations are numerically fitted with polynomial given by

$$\ln Y_{fit} = \sum_{n=0}^{4} a_n (\ln X)^n \tag{7}$$

where Y_{fit} are fitted values corresponding to numerical results Y. In Figs.-1,2,3, respectively, the plots of dimensionless moment of inertia (I) versus dimensionless tidal deformability (Λ), dimensionless moment of inertia (I) versus dimensionless quadrupole moment (Q) and dimensionless quadrupole moment (Q) versus dimensionless tidal deformability (Λ) are shown for APR, BSk22, BSk24 and BSk26 EoSs corresponding to maximum masses. The fitting curves (black dashed), given by Eq.(7), for the universal relation for these are also shown. It may be inferred that universal relations strongly emerge for these cases.

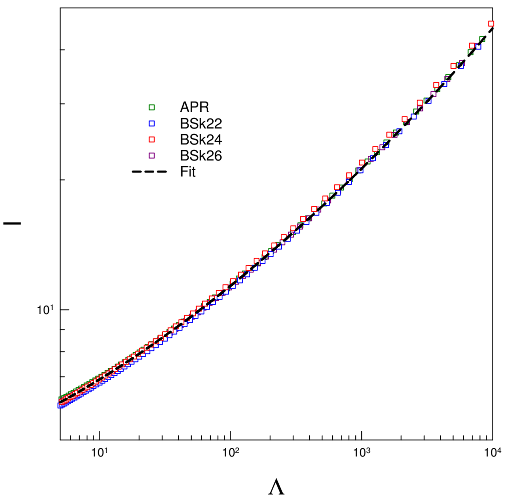

FIG. 1: Fitting curve (black dashed), given in Eq.(7), and numerical results (points) of the universal relation for dimensionless moment of inertia (I) vs dimensionless tidal deformability (Λ) for APR, BSk22, BSk24 and BSk26 EoSs.

Summary and Conclusion

We have used canonical (APR) and Brussels-Montreal Skyrme (BSk22, BSk24, BSk26) EoSs describing hadronic matter of NS to explore its properties such as mass (M), radius (R), Love number (k_2), dimensionless tidal deformability (Λ), dimensionless quadrupole moment(Q) and dimensionless moment of inertia (I) within a perturbative approach. Several relationships between these computed values have been examined. These relationships have been observed to be nearly independent of the specifics of the interior structure of NS. This notion of universality states that, even though certain quantities in a universal relation may not be observable, measurements of one quantity of that relation invariably reveal information about the others. These can be used to test General

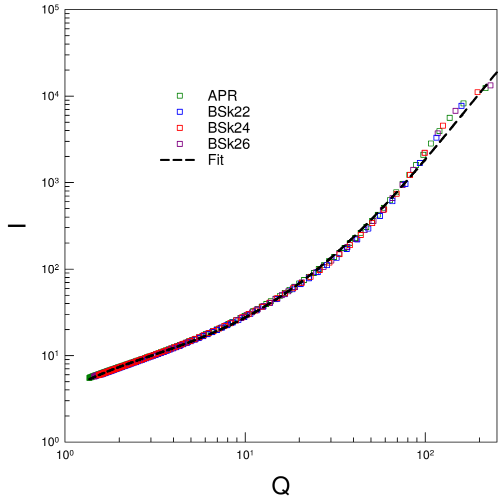

FIG. 2: Fitting curve (black dashed), given in Eq.(7), and numerical results (points) of the universal relation for dimensionless moment of inertia (I) vs dimensionless quadrupole moment (Q) for APR, BSk22, BSk24 and BSk26 EoSs.

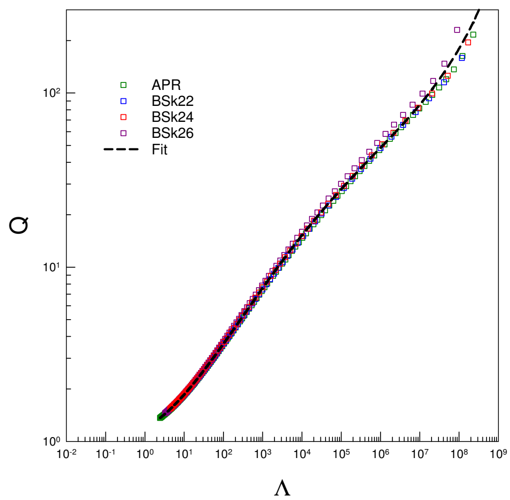

FIG. 3: Fitting curve (black dashed), given in Eq.(7), and numerical results (points) of the universal relation for dimensionless quadrupole moment (Q) vs dimensionless tidal deformability (Λ) for APR, BSk22, BSk24 and BSk26 EoSs.

Relativity independently of nuclear structure, quantify spin in binary inspirals by breaking degeneracies in GW detection and assess deformability of NSs by moment of inertia measurements.

Acknowledgments

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