Induced three-neutron interaction in dilute neutron matter

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Introduction

A microscopic study of the superfluidity in the interior of the neutron stars is essential to understand the observed phenomena such as pulsar glitches and the thermal evolution of the star. The neutron star crust consists of less dense superfluid neutron gas and degenerate electron gas interspersed among a lattice of bound nuclear clusters. Though the composition of the inner crust is well-established, a complete microscopic understanding of the physics of the neutron star crusts has not been achieved due to the uncertainties in the nuclear interactions.

A starting point for a microscopic study of the inner crusts of the neutron star is to ignore the presence of lattice and approximate the system with a gas of superfluid neutrons. The equation of state (EoS) of the neutron matter at very low densities is given by the $k_F a$-expansion, where k_F is the Fermi momenta and a is the scattering length. The traditional approach to obtain the scattering length from the phenomenological nucleon-nucleon (NN) interactions is to resum the ladder diagrams via the Brueckner theory. An alternative approach is to use the renormalization group based low-momentum interactions ($V_{\text{low}-k}$) or the similarity renormalization group (SRG) interactions, which decouple the low- and high-momentum regions and soften the interactions, thus allowing one to perform perturbative many-body calculations [2]. The results obtained from such calculations depend on the renormalization scale Λ which indicates the missing higher-body and medium corrections.

In [1], using the $V_{\text{low}-k}$ interaction with a density-dependent cutoff $\Lambda = f k_F$, where f is a scale factor, the EoS of the dilute neutron matter was obtained using the Bogoliubov many-body perturbation theory (BMBPT). The results showed a small residual cutoff dependence, indicating missing corrections. It is well known that as one decreases the cutoff, the contribution of the induced three-body (NNN) forces increases. Unlike with $V_{\text{low}-k}$, it is possible to evolve higher-body interactions consistently with the SRG formalism. So far, mostly the Jacobi partial-wave basis has been used for the momentum space SRG evolution of the NNN interactions, which has been applied to both finite and infinite matter [3]. In [4], the hyperspherical partial-wave (HPW) momentum basis was used for the evolution and applied successfully to the triton.

Our aim is to obtain the induced NNN interactions in the HPW basis via the SRG formalism and to study its effect on the EoS of the dilute neutron matter.

Formalism

Since we are interested in the dilute neutron matter, where the s-wave plays a dominant role, we restrict ourselves to the NNN interaction induced by the s-wave. Thus, we assume the Hamiltonian to be of the form

$$\begin{aligned} H_s &= T + V_s + W_s \\ &= \sum_{\mathbf{p}\sigma} \varepsilon_p a^\dagger_{\mathbf{p}\sigma} a_{\mathbf{p}\sigma} \\ &\quad + \sum_{1\dots4} V_{1234}(s) a^\dagger_{1\uparrow} a^\dagger_{2\downarrow} a_{4\downarrow} a_{3\uparrow} \qquad (1) \\ &\quad + \frac{1}{8} \sum_{\substack{1\dots6 \\ \sigma}} W_{123456}(s) a^\dagger_{1\sigma} a^\dagger_{2\bar{\sigma}} a^\dagger_{3\sigma} a_{6\sigma} a_{5\bar{\sigma}} a_{4\sigma} \,, \end{aligned}$$

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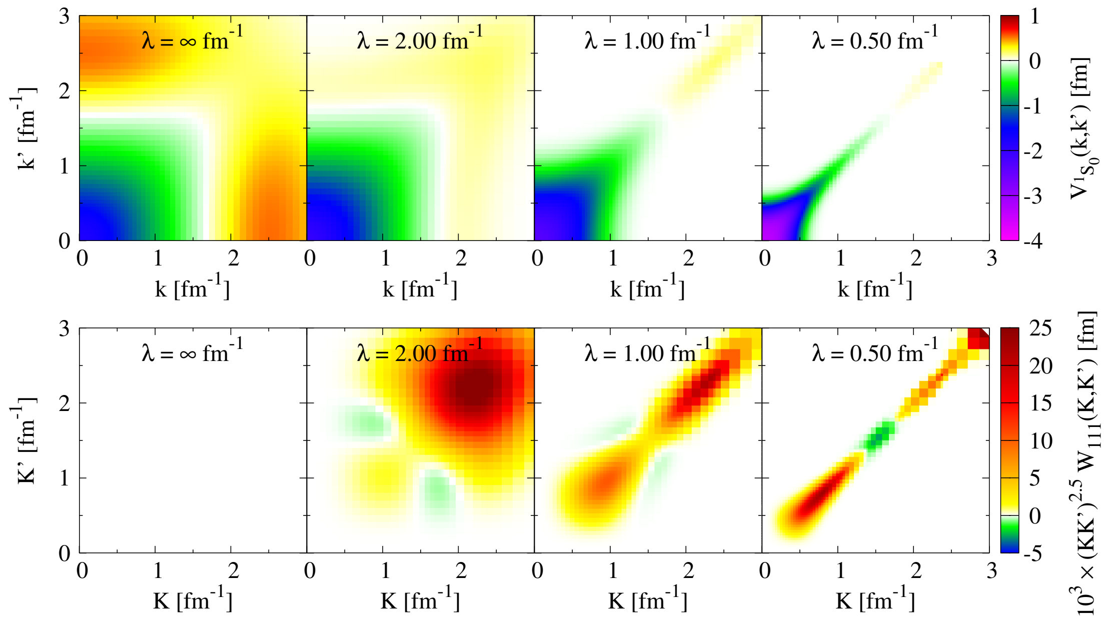

FIG. 1: Contour plots of the evolved NN and NNN-induced interactions for different SRG cutoffs.

where $i \equiv \mathbf{p}_i$, $\bar{\sigma} \equiv -\sigma$, and s is the SRG flow parameter which is usually replaced with the renormalization scale $\lambda \equiv s^{-1/4}$. W_{123456} is the antisymmetrized NNN matrix elements and $W_{123456}(0) = 0$ (no bare NNN interaction).

The SRG flow equation is given by

$$\frac{dH_s}{ds} = [\eta_s, H_s]\,, \tag{2}$$

where $\eta_s = [T, H_s]$ is the generator of the unitary transformation. Plugging in the Hamiltonian, we get the flow equations for the two- and three-body matrix elements in terms of the single-particle momenta. After rewriting them using the relative momenta, which are called the Jacobi momenta in the NNN case, we then project them on their respective partial wave basis.

Results and Discussion

The results presented are obtained using the SRG-evolved N^4LO chiral interaction. Fig. 1 shows the two-body s-wave matrix elements (top) and the induced NNN interaction in the dominant HPW channel (bottom) for different SRG cutoffs. It can be seen that as one evolves the NN interaction, a repulsive NNN interaction is induced. Also, as the cutoff decreases, both the two- and three-body matrix elements become band diagonal, and the region below the cutoff grows in magnitude as expected.

Work is in progress to compute the Hartree-Fock and Hartree-Fock-Bogoliubov contribution of the SRG-induced NNN interactions to the EoS of the dilute neutron matter.

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