Analysis of neutron star observables: A Bayesian Approach

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Introduction

Investigations of nucleonic matter is of great importance as it formulates the finite nuclei as well as the core of the neutron stars (NS), however it is quite challenging accounting to the strong interactions. Systems composed of degenerate gases and strongly interacting particles are further challenging to explore for the investigations of quantum correlations. NS are the perfect candidates to explore the dynamics of such systems as gravity compresses the matter upto one order of nuclear saturation density.

In this paper we present the bayesian analysis [1] and inference of neutron star observables within relativistic mean field realm for the mean field models given in our previous works [2–6]. The data within our own models is generated at the level of mean field theory and it is evident in previous works [2–6], that it sufficiently explains the nuclear observables.

Theoretical Formalism

The statistical analysis of the neutron star observables is conducted with bayesian technique [1], which provides the statistical inference by incorporating prior estimations into the analysis. It mainly utilises the Bayes theorem as [1],

$$P(\boldsymbol{\theta}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta})P(\boldsymbol{\theta})}{\mathcal{Z}}, \tag{1}$$

where D and $\boldsymbol{\theta}$ refers to the set of chosen fitting data and set of model parameters, respectively. $P(\boldsymbol{\theta}|D)$ indicates the posterior distribution of the chosen parameters, while $\mathcal{L}(D|\boldsymbol{\theta})$ signifies the likelihood function with $P(\boldsymbol{\theta})$ being the prior input for the prior knowledge of the model parameters and the normalization constant in the denominator ($\mathcal{Z}$) is the marginal likelihood or evidence which is just the renormalization of the probability. For a particular variable and/or parameter θ_i, the marginalized posterior distribution can be estimated as,

$$P(\theta_i|D) = \int P(\boldsymbol{\theta}|D) \prod_{k\neq i} d\theta_k. \tag{2}$$

For this study, in accordance with the existing literature, we are utilising the gaussian form of likelihood function [1], where the likelihood for measuring D is given by,

$$\mathcal{L}(D|\boldsymbol{\theta}) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{1}{2}\frac{(d-m(\theta))^2}{\sigma^2}\right), \tag{3}$$

where d and m are data and corresponding model values, respectively while σ is the uncertainty value. The evidence ($\mathcal{Z}$) is useful for comparison of different probabilistic models but in this work it is of less relevance for estimation of co-dependence of neutron star observables.

Results and Discussion

We have considered a wide range of nuclear models given in our previous studies [2–6] for the bayesian analysis of neutron star observables namely M_{max}, $\rho_{1.4}$, $R_{1.4}$, and $\Lambda_{1.4}$ which signifies maximum mass (in M⊙), density (in g/cm^3), radius of a neutron star with a mass of 1.4 M⊙ (in km), and dimensionless tidal deformability of 1.4 $M_\odot$ neutron star. These observables are crucial in understanding the

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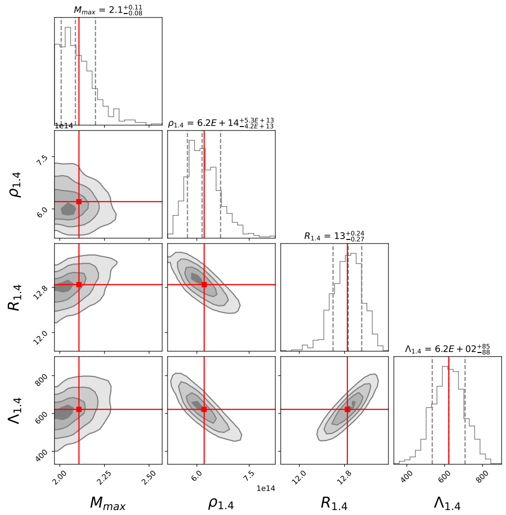

FIG. 1: Posterior distribution of the nuclear observables M_{max} maximum mass (in units of solar mass M⊙), $\rho_{1.4}$ density (in units of g/cm^3), $R_{1.4}$ radius (in units of km), and $\Lambda_{1.4}$ tidal deformability (dimensionless) of a neutron star.

properties of extreme objects, where M_{max} determines the boundary between neutron stars and black holes with implications for understanding equation of state for dense matter, $\rho_{1.4}$ provides insight into the composition and structure of neutron star cores, $R_{1.4}$ is a key parameter in determining the star's internal structure, cooling mechanisms, and potential for gravitational wave emission, and $\Lambda_{1.4}$ is a key observable for gravitational wave astronomy. All these parameters are essential in constraining the equation of state of dense matter and understanding the behavior of neutron stars in various astrophysical contexts. All of these observables (M_{max}, $\rho_{1.4}$, $R_{1.4}$, and $\Lambda_{1.4}$) were explicitly investigated in our previous works [2–6] for the purpose of the observed data required for the bayesian analysis. Fig.1 shows the posterior distribution of nuclear observables. The central values of observables are presented within the standard deviation range of 1σ (grey dashed vertical lines). The posterior distribution of M_{max}, $\rho_{1.4}$, $R_{1.4}$, and $\Lambda_{1.4}$ is centered around 2.1 M⊙, 6.2×10^{14} g/cm^3, 13 km, and 620, respectively with minimal amount of uncertainties which is consistent with our earlier findings and existing literature. Further the correlations amongst the observables is also shown in Fig.2. It can be observed that M_{max} shows weak correlations with other observables. It is evident from the Fig.2 that $\Lambda_{1.4}$ shows strong dependence on $R_{1.4}$ as can be observed from their strong positive correlations, as expected, which suggests that larger NS radii (stiffer equation of states) possibly leads to larger tidal deformabilities. Finally this powerful technique provides the results consistent with the existing literature.

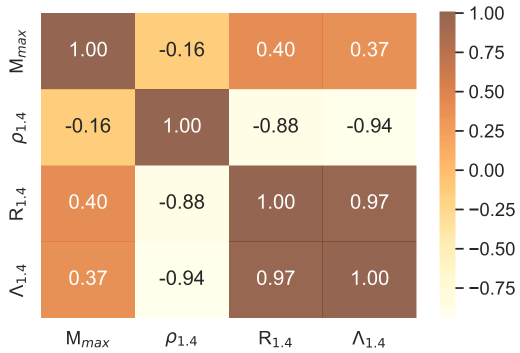

FIG. 2: Correlation coefficients for nuclear observables M_{max}, $\rho_{1.4}$, $R_{1.4}$, and $\Lambda_{1.4}$.

Acknowledgments

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