Diffraction-Like Pattern for Wigner distribution

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Introduction

The Wigner distribution provides insights into both momentum space variables and their position-space counterparts, making it a valuable tool for exploring the quantum system in phase space. Useful information about the spin and angular momentum correlations of quarks and gluons in the nucleon can be obtained from the Wigner distribution[1–3]. These distributions are defined in six-dimensional phase space and they reduce to well known GPDs and TMDs after certain phase-space reductions. This article reports on the behavior of quark and gluon Wigner distributions in boost-invariant longitudinal space for a strongly bound target state.

Wigner Distribution in QCD

In the context of Quantum Chromodynamics (QCD), Wigner distributions are quasi-probabilistic distributions that describe the combined position and momentum space distributions of quarks and gluons in a nucleon[4, 5]. The quark Wigner operator is defined as[1]

$$W^{[\Gamma]}(b_\perp, k_\perp, x) = \frac{1}{2}\int \frac{dz^-}{2\pi}\frac{d^2z_\perp}{(2\pi)^2} e^{i(xp^+z^- - k_\perp . z_\perp)}\bar{\psi}(y-\frac{z}{2})\Gamma\mathcal{W}\psi(y+\frac{z}{2})\Big|_{z^+=0},$$

and for gluon, it is defined as [3]

$$xW_{\lambda\lambda'}(x, k_\perp, \Delta_\perp, \sigma) = \int \frac{d\xi}{2\pi} e^{i\sigma\cdot\xi}\int \frac{dz^- d^2z_\perp}{2(2\pi)^3 p^+} e^{ik.z}\Big\langle p^+, \frac{\Delta_\perp}{2}, \lambda'\Big|\Gamma^{ij}F^{+i}\Big(-\frac{z}{2}\Big)F^{+j}\Big(\frac{z}{2}\Big)\Big|p^+, \frac{\Delta_\perp}{2}, \lambda\Big\rangle\Big|_{z^+=0}$$

where $x = \frac{k^+}{p^+}$ average longitudinal momentum fraction of parton, $\Delta_\perp$ is the momentum transfer from the target state in the transverse direction, $b_\perp$ is the 2-dimensional impact parameter space vector which is conjugate to $\Delta_\perp$. Γ represents a twist-two Dirac structure operator. For quarks, Γ is given by $\{\gamma^+, \gamma^+\gamma^5, i\sigma^{+j}\gamma^5\}$, while for gluons, it is defined as $\{\delta^{ij}, -i\epsilon_\perp^{ij}\}$. In longitudinal impact parameter space the Wigner distribution is defined as[2, 3],

$$\rho^{[\Gamma]}(x, \sigma, \Delta_\perp, k_\perp; S) = \int_0^{\xi_{max}} \frac{d\xi}{2\pi} e^{i\sigma\cdot\xi} W^{[\Gamma]}(x, \xi, \Delta_\perp, k_\perp; S)$$

The skewness (ξ) represents the longitudinal momentum transferred to the target state, given by $-\frac{\Delta^+}{2P^+}$. The upper cut off ξ_{max} is defined as

$$\xi_{max} = \frac{-t}{2m^2}\Big(\sqrt{1+\frac{4m^2}{-t}} - 1\Big),$$
$$\text{and } t = -\frac{4\xi^2m^2 + \Delta_\perp^2}{1-\xi^2}.$$

In the absence of longitudinal momentum transfer to the target state, the total transverse momentum transfer is $t = -\Delta_\perp^2$.

Results and Discussions

The Wigner distribution of unpolarized quark/gluon within an unpolarized target state can further be rewritten as the Fourier

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transfer of GTMD [4, 5]

$$\rho_{UU}(x,\sigma,\Delta_\perp,k_\perp) = \int_0^{\xi_{max}} \frac{d\xi}{2\pi} e^{i\sigma\cdot\xi} \frac{1}{\sqrt{1-\xi^2}} F_{1,1}$$

$$\rho^g_{UU}(x,\sigma,\Delta_\perp,k_\perp) = \int_0^{\xi_{max}} \frac{d\xi}{2\pi} e^{i\sigma\cdot\xi} \sqrt{1-\xi^2} S^{0,+}_{1,ia}$$

where the analytical expression for GTMDs have been obtained in our recent work [4, 5]

$$F_{1,1} = \frac{\alpha(1-\xi^2)}{2(1-x)^3}\Big[4(1+x^2-(3+x^2)\xi^2 + 2\xi^4)k_\perp^2 + 4m^2(1-x)^4 + 4(1-x)\xi (1+x^2-2\xi^2)k_\perp.\Delta_\perp - (1-x)^2(1+ x^2-2\xi^2)\Delta_\perp^2\Big]$$

$$S^{0,+}_{1,1a} = -\frac{\alpha_g}{2x_g}\Big[4m^2((1-x_g)^2-\xi^2)^2 + (1+ x_g^2-\xi^2)(4(1-\xi^2)k_\perp^2 - x_g^2\Delta_\perp^2 + 4x_g\xi k_\perp\cdot\Delta_\perp)\Big]$$

where the constants α, α_g are defined as follows:

$$\alpha = \frac{N}{D(q_\perp, y)D^*(d'_\perp, x')(x^2-\xi^2)}$$

$$\alpha_g = \frac{N\sqrt{1-\xi^2}}{D(q_{\perp g}, y_g)D^*(q'_{\perp g}, x'_g)x_g} \frac{1}{((1-x_g)^2-\xi^2)^{\frac{3}{2}}}$$

The normalization constant (N) for quark is $\frac{g^2C_f}{2(2\pi)^3}$, while for gluon, it is $\frac{g^2C_f}{2(2\pi)^2}$.

The oscillatory behavior in our results mirrors the single-slit diffraction pattern seen in optics, with σ being comparable to the slit-screen distance and skewness (ξ_{max}) to the slit width. We noticed that as $-t$ increases, the central peak becomes narrower, similar to optical diffraction where the slit width is inversely proportional to the width of central maxima.

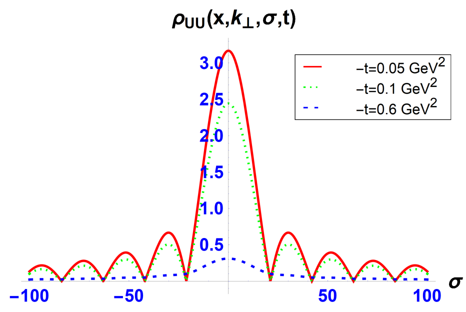

FIG. 1: Quark Wigner distribution in σ-space for various values of $-t$.

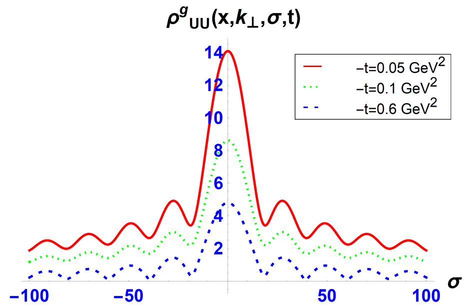

FIG. 2: Gluon Wigner distribution in σ-space for various values of $-t$.

Conclusion

The Wigner distribution of quarks and gluons exhibits a diffraction-like pattern. As energy transfer to the system $(-t)$ increases, the width of central peak of the Wigner distribution decreases.

References

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