Parton distribution functions of pion and kaon

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I. INTRODUCTION

The multidimensional structure of the elementary particles is one of the primary aspects for theoretical and experimental physicists. The parton distribution functions (PDFs) are the simplest distribution functions used to reveal the one-dimensional structure in terms of longitudinal momentum fraction (x) of the valence quark [1]. PDFs are the probability densities of finding a parton within a hadron. In case of spin-0 mesons, there is only one unpolarized $f(x)$ PDF at the leading twist wheras in the case of spin-1/2 nucleons there are 3 PDFs. The PDFs can be extracted from the deep inelastic scattering (DIS) processes. In this work, we have calculated the pion and kaon valence quark PDFs in the light-front quark model by solving the quark-quark correlator. We have also evolved these PDFs to compare our results with the available experimental data.

II. LIGHT-FRONT QUARK MODEL

The meson light-front (LF) minimal Fock-state representation with momentum P can be written as

$$\begin{aligned} |M(P,S_Z)\rangle &= \sum_{\lambda_i,\lambda_j}\int \frac{\mathrm{d}x\,\mathrm{d}^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} \\ &\times\ \Psi_{S_Z}(x,\mathbf{k}_\perp,\lambda_i,\lambda_j)|x,\mathbf{k}_\perp,\lambda_i,\lambda_j\rangle. \end{aligned}$$

$P=(P^+,P^-,P_\perp)$ is the four-vector average momentum of the hadron. $x=\frac{k^+}{P^+}$ and $\mathbf{k}_\perp$ are the longitudinal momentum fraction and the transverse momentum of the active quark respectively. λ_i and λ_j are the helicities of the constituent quark and anti-quark respectively. The spin of the meson, in the present case, is given by $S_Z=0$. The LF meson wave function $\Psi_{S_Z}(x,\mathbf{k}_\perp,\lambda_i,\lambda_j)$ with various spin and helicity projections λ can be expressed as

$$\begin{aligned} \Psi_{S_z}(x,\mathbf{k}_\perp,\lambda_i,\lambda_j) &= \Xi_{S_z=0}(x,\mathbf{k}_\perp,\lambda_i,\lambda_j) \\ &\times\ \phi(x,\mathbf{k}_\perp), \end{aligned} \quad (1)$$

where $\Xi_{S_z}(x,\mathbf{k}_\perp,\lambda_i,\lambda_j)$ and $\phi(x,\mathbf{k}_\perp)$ are the spin and momentum space wave functions of the meson respectively [1].

The momentum space wave function can be written using the Brodsky-Huang-Lepage (BHL) prescription as

$$\phi(x,\mathbf{k}_\perp) = N_0\,\exp\Bigg[-\frac{\frac{\mathbf{k}_\perp^2+m_q^2}{x}+\frac{\mathbf{k}_\perp^2+m_{\bar{q}}^2}{1-x}}{8\beta^2} - \frac{(m_q^2-m_{\bar{q}}^2)^2}{8\beta^2\left(\frac{\mathbf{k}_\perp^2+m_q^2}{x}+\frac{\mathbf{k}_\perp^2+m_{\bar{q}}^2}{1-x}\right)}\Bigg]. \quad (2)$$

The momentum space wave function is normalized as

$$\int \frac{dx\,d^2\mathbf{k}_\perp}{2(2\pi)^3}\,|\varphi(x,)|^2 = 1\,, \quad (3)$$

where $N_0 = Nexp\left(\frac{m_q^2+m_{\bar{q}}^2}{4\beta}\right)$. m_q and $m_{\bar{q}}$ are the quark and anti-quark masses of the meson. β is the harmonic scale parameter, set to be 0.410 GeV and 0.405 GeV for pion and kaon respectively. N is the normalization constant. The pion and kaon bound state masses are defined as

$$M^2_{\pi(K)} = \frac{m_q^2+\mathbf{k}_\perp^2}{x}+\frac{m_{\bar{q}}^2+\mathbf{k}_\perp^2}{1-x}.$$

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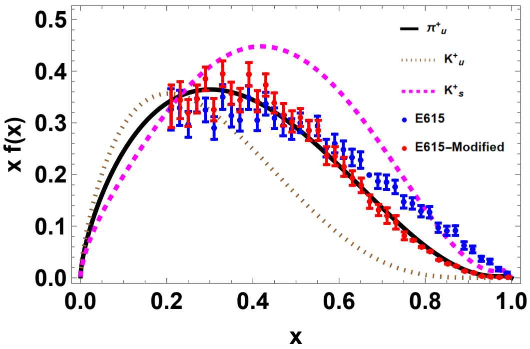

FIG. 1: u-quark and s-quark PDFs of pion and kaon have been plotted with respect to x at $Q^2 = 16$ GeV^2 evolved from an initial scale of Q_0^2. Our results have also been compared with modified FNAL-E615 data [2] and FNAL-E615 data [3]

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III. PARTON DISTRIBUTION FUNCTION

The PDFs can be used to access the probability of finding the valence quark in pion and kaon with longitudinal momentum fraction x. At a fixed LF time τ, the correlator of the leading twist unpolarized PDF is defined as [1]

$$f(x) = \frac{1}{2}\int \frac{dz^-}{4\pi} e^{ik^+z^-/2} \langle M(P,S_Z)|\bar{\psi(0)}\gamma^+\psi(z^-)|M(P,S_Z)\rangle,$$

where $z = (z^+, z^-, z_\perp)$ is the position four vector and $\psi(z)$ is the quark field operator. In this work, we have taken the Wilson line as unity.

The overlap form of the PDF, $f(x)$ in the form of LF wave function of Eq. (1) is found to be

$$f(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\Big[|\Psi(x,\mathbf{k}_\perp,\uparrow,\uparrow)|^{\mathbf{2}} + |\Psi(x,\mathbf{k}_\perp,\uparrow,\downarrow)|^{\mathbf{2}} + |\boldsymbol{\Psi}(\mathbf{x},\mathbf{k}_\perp,\downarrow,\uparrow)|^{\mathbf{2}} + |\Psi(x,\mathbf{k}_\perp,\downarrow,\downarrow)|^{\mathbf{2}}\Big].$$

IV. RESULTS

For numerical predictions, we have taken the quark masses $m_u = 0.2$ GeV and $m_s = 0.556$ GeV [1]. The PDFs of our result have been plotted with respect to longitudinal momentum fraction (x) in Fig. 1 at $Q^2 = 16$ GeV^2. The initial model scale of LFQM is $Q_0^2 = 0.20$ GeV^2. The pion u-quark PDF has been found to be consistent with modified FNAL-E615 data [2], while deviating from FNAL-E615 data [3] after $x = 0.6$. These evolutions are carried out by next to leading order (NLO) Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations. In case of kaon, the s-quark carries a larger momentum fraction than the u-quark. This indicates that the higher the mass of the quark, the larger would be the momentum fraction, which can be seen from Fig. 1. The average valence u-quark PDF contribution $\langle x\rangle$ is found to be 0.50 and 0.37 for pion and kaon respectively. While for the case of s-quark, it is found to be 0.63 at the model scale. However at $Q^2 = 1.64$ GeV^2, the $\langle x\rangle$ of u-quark of pion is found to be 0.17. The inverse moment $\langle x^-1\rangle$ of pion is found to be 2.79, which matches with other model predictions.

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