S-wave and P-wave masses of $\Omega_{QQ}(Q \in \{c, b\})$ baryon using relativistic quark model.

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Introduction

Recently, there has been significant experimental progress in studying hadrons with heavy quarks. Quantum Chromodynamics(QCD) predicts the existence of heavy baryons, which has made their study a major focus in the field of particle physics. The experimental studies of doubly heavy omega baryons are primarily conducted by the SELEX, LHCb and BELLE collaborations[1], while theoretical studies utilize various models, including the non-relativistic quark model[2], lattice QCD[3] and other potential models. Currently, there is no experimental data available for doubly heavy baryons in the Ω sector. The production of these baryons is particularly challenging, which has contributed to the lack of empirical information. As a result, theoretical studies play a crucial role in advancing our understanding of these particles. In the present study, we have calculated the mass spectra of the Ω_{cc} and Ω_{bb} baryon using the relativistic quark model.

Theoretical Model

To study the mass spectra of doubly heavy Ω_{QQ} baryon in relativistic approach, we have employed a relativistic Dirac formalism with non-coloumbic linear potential given by,

$$V(r) = Ar + (\lambda_i \cdot \lambda_j)\alpha_s \quad (1)$$

Where, A represents the strength of the potential and $(\lambda_i \cdot \lambda_j)\alpha_s$ is a constant negative potential depth. We have utilized the Dirac equation(with $\hbar = c = 1$)within the relativistic framework[4].

$$[E^{Dir} - \vec{\alpha} \cdot \vec{P} - m_Q\beta - V(r)]\psi(\vec{r}) = 0 \quad (2)$$

Consequently, the equivalent energy eigenvalue is given by,

$$\epsilon^{Dir} = (E^{Dir} - m_Q - V_0)[(2A)^{-2}(E^{Dir} + m_Q)]^{\frac{1}{3}}. \quad (3)$$

where, E^{Dir} is the Dirac independent-quark binding energy of the QQq bound state system. The masses of three body baryon system can be given by,

$$M^{Dir} = E_d^{Dir} + E_q^{Dir} + \langle V_{SD} \rangle \quad (4)$$

where, $\langle V_{SD} \rangle$ stands for spin-dependent interaction potential, which includes spin-spin (V_{jj}), spin-orbit (V_{LS}) and spin-tensor (V_T) interactions for the better understanding of the splitting between states with different quantum numbers. The spin-spin(j-j) interaction can be express as[5]

$$\langle V_{jj} \rangle = \frac{\sigma \langle j_d j_q JM | \hat{j}_d \hat{j}_q | j_d j_q JM \rangle}{(E_d + M_d)(E_q + m_q)} \quad (5)$$

where σ is the j-j coupling constant. $\langle j_d j_q JM | \hat{j}_d \hat{j}_q | j_d j_q JM \rangle$ contains the square of the Clebsch-Gordan coefficient. The spin-orbit interaction and spin-tensor interactions are express as[5]

$$\langle V_{LS} \rangle = -\frac{\alpha_s}{3} \frac{N_d^2 N_q^2}{(E_d + m_d)(E_q + m_q)} \frac{1}{r}$$
$$\otimes[\vec{r} \times (\hat{P}_d - \hat{P}_q) \cdot (\sigma_d + \sigma_q)](D_0'(r) + 2D_1'(r))$$
$$+[\vec{r} \times (\hat{P}_d + \hat{P}_q) \cdot (\sigma_d - \sigma_q)](D_0'(r) - D_1'(r)) \quad (6)$$

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and

$$\langle V_T \rangle = \frac{2\alpha_s}{3} \frac{N_d^2 N_q^2}{(E_d+m_d)(E_q+m_q)} \otimes ((\frac{D_1^{''}(r)}{3} - \frac{D_1^{'}(r)}{3r}) S_{dq}) \quad (7)$$

Result and conclusion

In this present work, we have predicted the masses of radially and orbitally excited states of the Ω_{cc} and Ω_{bb} baryons using the relativistic quark model. To calculate the mass spectra, we have solve the Dirac equation with a non-Coulombic linear potential. The computed masses for the ground and excited states of the Ω_{cc} and Ω_{bb} baryons are presented in Tables I and II respectively. We compare our results with other available theoretical predictions. For the Ω_{cc} baryon, the predicted masses for the ground states are 3784 MeV for $|1S, 1/2^+\rangle$ and 3859 MeV for $|1S, 3/2^+\rangle$, which are in good agreement with values reported in Refs.[6] and [7]. For the Ω_{bb} baryon, our results show a mass of 10381 MeV for $|1S, 1/2^+\rangle$ and 10403 MeV for $|1S, 3/2^+\rangle$, aligning closely with values reported in Ref.[6]. Hence, We have effectively predicted the S-wave and P-wave masses of the Ω_{cc} and Ω_{bb} baryons, showing consistency with existing theoretical results. Our approach provides a reliable basis for future studies of these heavy baryon states.

TABLE I: Masses of Ω_{cc} baryon(in Mev).

State	Present	[6]	[7]	[8]	[9]
$|1S, 1/2^+\rangle$	3784	3778	3766	3719	3736
$|1S, 3/2^+\rangle$	3859	3872	3833	3847	3837
$|2S, 1/2^+\rangle$	4121	4075	-	4041	4078
$|2S, 3/2^+\rangle$	4185	4174	-	4096	4110
$|1P, 1/2^-\rangle$	4294	4217	4111	-	4011
$|1P, 3/2^-\rangle$	4304	4325	4144	3947	4004
$|1P, 1/2^-\rangle$	4232	4208	4145	-	4014
$|1P, 3/2^-\rangle$	4236	4252	4177	-	4007
$|1P, 5/2^-\rangle$	4241	4303	4232	4153	3998
$|2P, 1/2^-\rangle$	4601	-	-	-	4253
$|2P, 3/2^-\rangle$	4604	-	-	4259	4248
$|2P, 1/2^-\rangle$	4555	-	-	-	4256
$|2P, 3/2^-\rangle$	4563	-	-	-	4251
$|2P, 5/2^-\rangle$	4564	-	-	4247	4244

TABLE II: Masses of Ω_{bb} baryon(in Mev).

State	Present	[6]	[7]	[9]
$|1S, 1/2^+\rangle$	10381	10359	10438	10357
$|1S, 3/2^+\rangle$	10403	10389	10460	10406
$|2S, 1/2^+\rangle$	10638	10610	-	10646
$|2S, 3/2^+\rangle$	10658	10645	-	10662
$|3S, 1/2^+\rangle$	10847	10806	-	10859
$|3S, 3/2^+\rangle$	10865	10843	-	10866
$|1P, 1/2^-\rangle$	10805	10804	10762	10580
$|1P, 3/2^-\rangle$	10810	10821	10768	10578
$|1P, 1/2^-\rangle$	10779	10771	10778	10581
$|1P, 3/2^-\rangle$	10782	10785	10784	10579
$|1P, 5/2^-\rangle$	10784	10798	10795	10576
$|2P, 1/2^-\rangle$	11092	-	-	10796
$|2P, 3/2^-\rangle$	11094	-	-	10795
$|2P, 1/2^-\rangle$	11073	-	-	10797
$|2P, 3/2^-\rangle$	11076	-	-	10796
$|2P, 5/2^-\rangle$	11077	-	-	10794

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