Strangeonium spectra in Regge phenomenology

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Introduction

Mesons consisting only of u, d, and s quarks are classified as light mesons. In this study, we focus on a particular quarkonium system known as strangeonium ($s\bar{s}$) states, which are bound states of the s-quark and anti-s quark, occupying an intermediate position between the lighter u and d quarks and the heavier charm and bottom quarks. The study of strangeonium states is closely connected to the investigation of non-$q\bar{q}$ states, such as glueballs, hybrids, and tetraquarks, which share the same quantum numbers as conventional $q\bar{q}$ mesons. A detailed understanding of these conventional $q\bar{q}$ states is essential to experimentally confirm the presence of non-$q\bar{q}$.

The latest Particle data group (PDG) [1] lists a limited number of experimentally confirmed strangeonium resonances that are widely accepted as pure $s\bar{s}$ states. Aside from certain low-lying $1P$ and $1D$ states, many of the excited strangeonium states have yet to be observed. The BESIII experiment provides a significant opportunity to study these states, as it both verifies previously observed $s\bar{s}$ resonances and identifies new ones through the decays of J/ψ and $\psi(2S)$ [2, 3]. Additionally, the forthcoming PANDA experimental facility is actively engaged in the search for these light mesons [4]. However, the quantum numbers of several of these $s\bar{s}$ states remain unresolved, emphasizing the need for continued investigation and verification.

Apart from the experimental study, a wealth of literature exists on the spectroscopy of light mesons, utilizing a variety of theoretical methods [5–8]. The objective of this study is to systematically analyze $s\bar{s}$ meson and determine the mass spectra. The theoretical framework employed is the well-established Regge phenomenology.

Theoretical Framework

Assuming linear Regge trajectories for light mesons, the most general expression for these trajectories can be formulated as follows [9–12],

$$J = \beta(0) + \beta^{'} M^2, \tag{1}$$

here, $\beta(0)$ and $\beta^{'}$ denote the intercept and slope of the trajectory, respectively. For a meson multiplet, the Regge parameters corresponding to different quark constituents are related by the following expressions [9–14],

$$\beta_{i\bar{i}}(0) + \beta_{j\bar{j}}(0) = 2\beta_{i\bar{j}}(0), \tag{2}$$

$$\frac{1}{\beta^{'}{}_{i\bar{i}}} + \frac{1}{\beta^{'}{}_{j\bar{j}}} = \frac{2}{\beta^{'}{}_{i\bar{j}}}, \tag{3}$$

here, i and j denote quark flavors. By solving relations (1), (2), and (3), we obtain an expression involving Regge slopes and meson masses, given as follows,

$$\begin{aligned}\frac{\beta^{'}_{j\bar{j}}}{\beta^{'}_{i\bar{i}}} &= \frac{1}{2M^2_{j\bar{j}}} \times [(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}}) \\ &\pm \sqrt{(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}})^2 - 4M^2_{i\bar{i}}M^2_{j\bar{j}}}].\end{aligned} \tag{4}$$

By utilizing Eq. (4), we can derive the high-power mass relations for mesons, which is expressed as follows,

$$\frac{\beta^{'}_{j\bar{j}}}{\beta^{'}_{i\bar{i}}} = \frac{\beta^{'}_{k\bar{k}}}{\beta^{'}_{i\bar{i}}} \times \frac{\beta^{'}_{j\bar{j}}}{\beta^{'}_{k\bar{k}}}, \tag{5}$$

where k represents any quark flavor. Now, Eqs. (4) and (5) provide additional relation

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*Electronic address: `juhioudichhya01234@gmail.com` between meson masses of different flavors, allowing for the determination of the ground state masses of $s\bar{s}$. Details of this study is available in Ref. [10]. For higher excited states, Regge slopes are computed for various trajectories. Based on the quark composition of strangeonium, by substituting $i = n$ and $j = s$ into Eq. (4), we can yield the Regge slope values, $\beta^{'}$ for different trajectories. Additionally, from Eq. (1), another relation involving meson masses and the Regge slope can be derived, expressed as follows,

$$M_{J+1} = \sqrt{M_J^2 + \frac{1}{\beta^{'}}}. \tag{6}$$

Thus, using the extracted Regge slope values, from the above equation the higher excited state masses along the Regge trajectories can be determined.

Results and Discussion

TABLE I: Excited state masses of strangeonuim in the (J, M^2) plane (in MeV).

$N^{2S+1}L_J$	This work	PDG [1]	[8]
1^1S_0	695.80±0.0		743
1^3S_1	1005.63±0.00	1019.46±0.016	1017
1^1P_1	1402.01±0.78	1416±8	1462
1^3P_0	1392.00±0.00	$1350 \pm 9^{+12}_{-2}$ [3]	1373
1^3P_1	1514.90±0.00		1492
1^3P_2	1518.70±17.67	1517.4±2.5	1513

The ground and excited state masses calculated by our model, presented in Table I, are compared with experimental data and theoretical predictions from Ref. [8]. Our model's predictions for ground state masses are consistent with both experimental measurements and theoretical values. As we go for higher excited states, the $h_1(1415)$ resonance, with a measured mass of 1416 ± 8 MeV and $J^P = 1^+$, is identified as a likely 1^1P_1 $s\bar{s}$ state. Our predicted mass of 1402.01 ± 0.78 MeV closely matches this, differing by only 14 MeV.

Similarly, the $f_2^{'}(1525)$ resonance, identified as the 1^3P_2 $s\bar{s}$ state, aligns well with our predicted mass of 1518.70 ± 17.67 MeV, differing by just 1 MeV. Although the 1^3P_0 $s\bar{s}$ state remains challenging, the $f_0(1370)$ meson, with a mass range of 1200-1500 MeV as listed by the PDG and a recent BESIII measurement of $1350 \pm 9^{+12}_{-2}$ MeV [3], is consistent with our model, suggesting it could be a strong candidate for the 1^3P_0 $s\bar{s}$ state. We anticipate that our predicted results will offer valuable insights for future experimental investigations to identify missing excited $s\bar{s}$ states.

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