Ground state masses of all strange tetraquark using Regge Phenomenology

Vandan Patel[1],* Juhi Oudichhya[1] and Ajay Kumar Rai[1]
[1] Department of Physics, Sardar Vallabhbhai
National Institute of Technology, Surat-395007, Gujarat, India

Introduction

Since the discovery of the first tetraquark candidate, the $X(3872)$ state, in 2003, a new domain of high-energy physics has emerged. Despite the success of the constituent quark model, numerous unconventional states that do not align with it have been both experimentally observed and theoretically predicted [1–9]. The leading explanations for these unusual resonances include hadronic molecules, tetraquarks, pentaquarks, hybrids, and more [3–5].

Although most tetraquark candidates identified experimentally contain at least one heavy quark, various theoretical studies in the past decade have proposed predictions for all-light tetraquarks. Among these, the $f_0(500)$ (previously known as σ), $a_0(980)$, and $f_0(980)$ have stood out as strong candidates for all-light tetraquarks [10].

Wei et al. [11], using a quasilinear Regge trajectory approach, derived several significant mass relations, such as quadratic mass equalities, linear mass inequalities, and quadratic mass equalities for hadrons. In this study, we apply the same Regge phenomenology under the assumption of linear Regge trajectories. We extract relationships between the intercept, slope ratios, and tetraquark masses in the (J, M^2) plane. Utilizing these relations, we have computed the ground state masses of all strange tetraquarks $ss\bar{s}\bar{s}$.

Theoretical Framework

Regge theory is one of the simplest and most practical phenomenological approaches for examining hadron spectroscopy. Various concepts were developed to understand Regge trajectories, with Nambu's being the most straightforward. His theory proposed that linear Regge trajectories arise from the uniform interaction of a quark-antiquark pair within a strong flux tube.

So, the linear Regge trajectory equation is given by,

$$J = \frac{M^2}{2\pi\sigma} + c'' \quad \ldots (1)$$

As a result, J and M^2 have a linear relationship with one another. The plots of hadron Regge trajectories in the (J, M^2) plane are referred to as Chew-Frautschi plots [].

So, from quasilinear Regge theory ansatz, the relationship between total angular quantum number (J) and mass (M) of hadron is given by,

$$J = \alpha(M) = a(0) + \alpha' M^2 \quad \ldots (2)$$

The Regge parameters (Regge slopes and Regge intercepts) for different quark constituents of a meson multiplet with spin-parity J^P can be connected by the following relations [12-17]:

$$a_{i\bar{i}}(0) + a_{j\bar{j}}(0) = 2a_{i\bar{j}}(0) \quad \ldots (3)$$

$$\frac{1}{\alpha'_{i\bar{i}}} + \frac{1}{\alpha'_{j\bar{j}}} = \frac{2}{\alpha'_{i\bar{j}}} \quad \ldots (4)$$

where quark flavours are denoted by (i) and (j). Using equations (3) and (4), we can derive

$$((M_{i\bar{i}} + M_{j\bar{j}})^2 - 4M_{i\bar{j}}^2) = \sqrt{\left(4M_{i\bar{j}}^2 - M_{i\bar{i}}^2 - M_{j\bar{j}}^2\right)^2 - 4M_{i\bar{i}}^2 M_{j\bar{j}}^2} \quad \ldots (5)$$

Above relation is the relationship between masses of three mesons. i.e. $M_{i\bar{i}}, M_{j\bar{j}}$ and $M_{i\bar{j}}$. Thus, by knowing two masses, a third mass can be predicted using equation (5).

In this work, we calculate the mass spectra of all light tetraquarks, treating them as bound states of two clusters (diquark and anti-diquark). The diquarks are considered as two coupled quarks without internal spatial excitation. Since a quark pair cannot form a color singlet, diquarks can only exist within hadrons and are used as an effective degree of freedom. Two quarks or two anti-quarks interact via gluon exchange to form the bound states known as diquark and anti-diquark.

---

*Electronic address: vandankp12998@gmail.com

Ground state masses of all strange tetraquark can be calculated by putting $i = [qq]$, $j = [ss]$ in equation (5),

$$\left(\left(M_{qq\overline{q}\overline{q}} + M_{ss\overline{s}\overline{s}}\right)^2 - 4M_{qq\overline{s}\overline{s}}^2\right) = \sqrt{\left(4M_{qq\overline{s}\overline{s}}^2 - M_{qq\overline{q}\overline{q}}^2 - M_{ss\overline{s}\overline{s}}^2\right)^2 - 4M_{qq\overline{q}\overline{q}}^2 M_{ss\overline{s}\overline{s}}^2} \quad \ldots(6)$$

By using above relation, we can calculate ground state masses of all strange tetraquark $ss\overline{s}\overline{s}$ for $J^P = 0^+, 1^+$ and 2^+.

Results and Discussion:

The ground state masses of all strange tetraquark ($ss\overline{s}\overline{s}$) for $J^P = 0^+, 1^+$ and 2^+ is calculated as shown in table 1. The calculated values are compared with Ref. [18] and with two meson threshold also.

Table 1. Mass spectra of $ss\overline{s}\overline{s}$ tetraquark (in GeV)

St-ate	J^P	$M_{(calc)}$ Calcu-lated Mass (in GeV)	[18]	Meson Threshold	M_{th} (Threshold mass) (GeV)
1^1S_0	0^+	2.103	2.293	$\eta'(958)$ $\eta'(958)$	1.916
1^3S_1	1^+	2.174	2.323	$\eta'(958)$ $\phi(1020)$	1.977
1^5S_2	2^+	2.508	2.378	$a_1(1260)$ $a_1(1260)$	2.460

The ground state tetraquark masses are comparable to the two-meson threshold. The calculated masses yield results comparable to those from other models and will aid in future experimental and theoretical studies of tetraquarks and other exotic hadrons.

Acknowledgment

Vandan Patel acknowledges the financial assistance by University Grant Commision (UGC) under the CSIRUGC Junior Research Fellow (JRF) scheme with Ref No. 231610186052. Juhi Oudichhya acknowledges the financial assistance by the Council of Scientific and Industrial Research (CSIR) under the Direct SRF fellowship scheme with file no. 09/1007 (18111)/2024-EMR-I.

References:

[1] Gang Yang, Jialun Ping, and Jorge Segovia, Symmetry, **12**, 1869 (2020).
[2] S. K. Choi et al., (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[3] D. P. Rathaud and A. K. Rai, Indian J. Phys. **95**, 2807-2828 (2021).
[4] D. P. Rathaud and A. K. Rai, Few-Body Syst, **60**, 1 (2019).
[5] H. X. Chen et al., Phys. Rept. **639**, 1 (2016).
[6] R. Tiwari, D. P. Rathaud, and A. K. Rai, AIP Conf. Proc., 2220, 140067 (2019).
[7] V. Debastiani and F. Navarra, Chin. Phys. C **43**, 013105 (2019).
[8] A. K. Rai et al., Indian J. Phys. A **80**, 387-392 (2006).
[9] A. K. Rai, et al., Nucl. Phys. A **782**, 406-409 (2007).
[10] R. J. Jaffe, Phys. Rev. D **15**, 267 (1977)
[11] K.-W. Wei, B. Chen, and X.-H. Guo, Phys. Rev. D **78**, 056005 (2008).
[12] D.-M. Li, B. Ma, Y.-X. Li, Q.-K. Yao, and H. Yu, Eur. Phys. J. C **37**, 323 (2004).
[13] L. Burakovsky, T. Goldman, and L. P. Horwitz, Phys. Rev. D 56, 7119 (1997); J. Phys. G **24**, 771 (1998).
[14] L. Burakovsky and T. Goldman, Phys. Lett. B **434**, 251 (1998).
[15] A. B. Kaidalov, Z. Phys. C **12**, 63 (1982).
[16] J. Oudichhya, K. Gandhi, and A. K. Rai, Phys. Rev. D **108**, 014034 (2023).
[17] J. Oudichhya, K. Gandhi, A.K. Rai, Phys. Rev. D **104**, 114027 (2021).
[18] F.X. Liu, M.S. Liu, X.H. Zhong, and Q. Zhao, Phys. Rev. D **103**, 016016 (2021).
[19] G. F. Chew and S. C. Frautschi, Phys. Rev. Lett. **7**, 394 (1961); **8**, 41 (1962).