Conductive properties of strongly interacting matter under nonextensivity

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Introduction

Transport coefficients play an important role in characterizing the development of the bulk matter generated in relativistic heavy-ion collisions. These coefficients offer a valuable understanding of the extent to which a system diverges from ideal hydrodynamics and discloses significant information about fluid dynamics and critical phenomena [1].

A range of effective QCD models have been used to study transport coefficients in great detail in recent years. However, the statistical approach used in many of the models is the standard Boltzmann-Gibbs (BG) statistics, whose validity in heavy ion collision experiments is found to be questionable [2]. To address these issues, Tsallis proposed nonextensive statistics as a generalization of the BG statistics by introducing a dimensionless q parameter to account for all potential variables that violate the assumptions of the standard BG statistics [3]. In the present study, we calculate the conductive transport coefficients of strongly interacting matter, namely, electrical conductivity σ_{el} and thermal conductivity κ using a nonextensive version of the Polyakov chiral SU (3) quark mean field ($q-$PCQMF) model.

$q-$PCQMF model

Tsallis framework is characterized by the substitution of the conventional exponential and logarithm function to $q-$modified forms defined as

$$exp_q(x) = [1 + (1-q)x]^{\frac{1}{1-q}}, \tag{1}$$

$$ln_q(x) = \frac{x^{1-q} - 1}{1-q}. \tag{2}$$

This leads to the modification of the thermodynamic potential density, which is given as

$$\Omega_q = \Omega_{q,\psi\bar{\psi}} - \mathcal{L}_M - \mathcal{V}_{vac} + \mathcal{U}(\Phi, \bar{\Phi}, T), \tag{3}$$

where $\Omega_{q,\psi\bar{\psi}}$ represents thermal contributions of quarks and antiquarks and is given by [4],

$$\Omega_{q,\psi\bar{\psi}} = \sum_{i=u,d,s} \frac{-\gamma_i k_B T}{(2\pi)^3} \int_0^\infty d^3k \{ln_q F_q^- + ln_q F_q^+\}, \tag{4}$$

with

$$F_q^- = 1 + e_q^{-3E^-} + 3\Phi e_q^{-E^-} + 3\bar{\Phi} e_q^{-2E^-}, \tag{5}$$

$$F_q^+ = 1 + e_q^{-3E^+} + 3\bar{\Phi} e_q^{-E^+} + 3\Phi e_q^{-2E^+}. \tag{6}$$

In the above, $E^- = (E_i^*(k) - \mu_i{}^*)/k_B T$ and $E^+ = (E_i^*(k) + \mu_i{}^*)/k_B T$ in which the in-medium energy of the quarks is given by $E_i^*(k) = \sqrt{m_i^{*2} + k^2}$ and $\mu_i{}^* = \mu_i - g_\omega^i \omega - g_\phi^i \phi - g_\rho^i \rho$.

In Eq. 3, $\mathcal{L}_M$ describes the self interaction of the scalar (σ, ζ, δ) and vector mesons (ω, ρ, ϕ), $\mathcal{V}_{vac}$ represents the vacuum energy, and $\mathcal{U}(\Phi, \bar{\Phi}, T)$ is the logarithmic Polyakov loop potential.

In the $q-$PCQMF model, the Fermi-Distribution functions also get modified to their $q-$form,

$$f_{q,i}(k) = \frac{\Phi e_q^q(-E^-) + 2\bar{\Phi} e_q^q(-2E^-) + e_q^q(-3E^-)}{[1 + 3\Phi e_q(-E^-) + 3\bar{\Phi} e_q(-2E^-) + e_q(-3E^-)]^q}, \tag{7}$$

$$\bar{f}_{q,i}(k) = \frac{\bar{\Phi} e_q^q(-E^+) + 2\Phi e_q^q(-2E^+) + e_q^q(-3E^+)}{[1 + 3\bar{\Phi} e_q(-E^+) + 3\Phi e_q(-2E^+) + e_q(-3E^+)]^q}. \tag{8}$$

Notably, we return to the standard (extensive) PCQMF model in the limit $q \to 1$, where

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the standard Fermi-Distribution functions are recovered. We apply the modified thermodynamic potential density within a quasiparticle approach in kinetic theory using the relaxation time approximation (RTA) to calculate the electrical conductivity σ_{el} and thermal conductivity κ. The mathematical expressions are given as:

$$\sigma_{el} = \frac{2N_c}{3T} \sum_{i=u,d,s} e_i^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3} \tau \left(\frac{k}{E_i^*}\right)^2 [f_{q,i}(1-f_{q,i}) + \bar{f}_{q,i}(1-\bar{f}_{q,i})], \quad (9)$$

$$\kappa = \frac{2N_c}{3T^2} \sum_{i=u,d,s} \int \frac{\mathrm{d}^3k}{(2\pi)^3} \tau \left(\frac{k}{E_i^*}\right)^2 [(E_i^* - h)^2 f_{q,i}(1-f_{q,i}) + (E_i^* + h)^2 \bar{f}_{q,i}(1-\bar{f}_{q,i})]. \quad (10)$$

Results and Discussions

In Fig. 1(a), we have plotted the scaled electrical conductivity σ_{el}/T as a function of temperature T at baryon chemical potential $\mu_B = 400$ MeV, isospin chemical potential $\mu_I = 40$ MeV, and strangeness chemical potential $\mu_S = 125$ MeV, for $q = 1, 1.05, 1.10$. We find that the value of σ_{el}/T increases with a rise in temperature. Moreover, its value increases when q goes over one. This may be due to the tendency of the quarks to become deconfined at lower temperatures for nonextensive systems. Fig. 1(b) shows the temperature variation of thermal conductivity κ/T^2 at $q = 1$, 1.05, and 1.10. We find that κ/T^2 rises monotonically with temperature. This may be due to the rising magnitude of the heat function h, leading to efficient transmission of heat within the QGP, thereby enhancing thermal conductivity. With regard to the effects of the q parameter, we observe that the value of κ/T^2 rises when $q > 1$, with a more pronounced effect in the high T range. It is noteworthy that both σ_{el}/T and κ/T^2 exhibit nonextensive effects only at elevated temperatures. In the low-temperature limit, the nonextensive signature vanishes as Ω_q (nonextensive) $\to \Omega$ (extensive) when $T \to 0$.

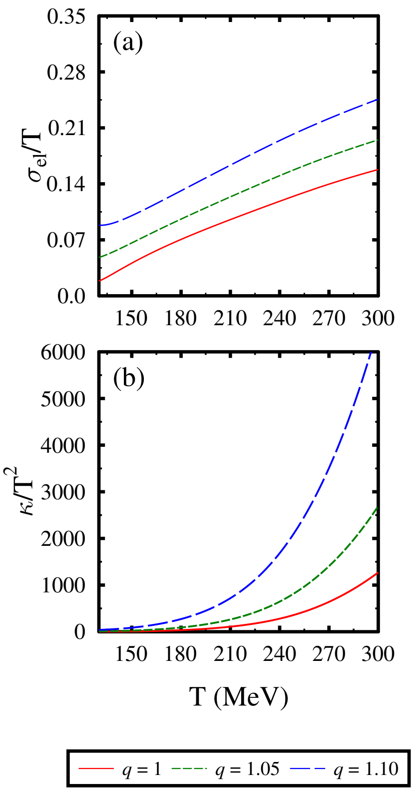

FIG. 1: Scaled electrical conductivity σ_{el}/T and thermal conductivity κ/T^2 plotted as a function of temperature T at $q = 1$, 1.05, and 1.10.

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