Dependence of μ_s on μ_q using Correlation of K^-/K^+ and $\bar{p}/p$ Ratios Through Interacting HRG Model

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Introduction

The study of ultra-relativistic heavy ion collisions provides a unique opportunity to investigate the properties of strongly interacting matter under extreme conditions of temperature and density. Particle ratios, such as $\bar{p}/p$ and K^-/K^+, serve as sensitive probes of the thermodynamic properties of the system created in these collisions. In this paper, we present a comprehensive study of these particle ratios, focusing on the role of strangeness conservation criteria in an ideal hadron resonance gas (HRG) model and an interacting HRG model based on the van der Waals equation of state. We explore the correlations between $\bar{p}/p$ and $\bar{K}/K$ ratios within the framework of the interacting HRG model and finally discuss the variation of strangeness chemical potential on baryon chemical potential [1, 3].

Model

Interacting Hadron Resonance Gas Model:

In ultra-relativistic nucleus-nucleus collisions, particle production ratios are well described by a grand canonical model, where temperature T and baryochemical potential μ_q are treated as independent parameters. The strange quark chemical potential μ_s is constrained by strangeness conservation. Thus, the ratios of antiparticles to particles depend on the fugacities of light and strange quarks, represented as μ_q/T and μ_s/T, respectively. In the van der Waals interacting hadron resonance gas model, the particle number density is adjusted from the ideal HRG model by incorporating an excluded volume parameter b and an attractive interaction parameter a [1].

$$n(\mu, T) = \frac{n^{\mathrm{id}}(T, \mu^*)}{1 + b n^{\mathrm{id}}(T, \mu^*)} \tag{1}$$

The number density $n_i(\mu_i, T)$ is calculated from the transcendental equation (1), where the ideal quantum gas number density $n_i^{id}(T, \mu_i)$ is given by:

$$n_i^{id}(T, \mu_i) = \frac{g_i}{2\pi^2} \int_0^\infty \frac{k^2 dk}{\exp\left(\frac{E_i - \mu_i}{T}\right) \pm \eta} \tag{2}$$

with $\eta = 0$ for Boltzmann statistics (operation $+$ is for fermions and $-$ is for bosons).

where μ^* is the modified chemical potential:

$$\mu^* = \mu - \frac{bnT}{1 - bn} + 2an \tag{3}$$

The chemical potentials for baryon number (B), strangeness (S), and electric charge (Q) are given by:

$$\mu_i = B_i \mu_B + S_i \mu_S \tag{4}$$

We employ the following parameterization for the baryon chemical potential and Temperature with same parameters. [4]:

$$\mu_B = \frac{c}{1 + d\sqrt{s_{NN}}}, \quad T = e - f\mu_B^2 - g\mu_B^4 \tag{5}$$

Correlation between K^-/K^+ and $\bar{p}/p$ ratios:

In heavy-ion collisions, particularly in high-energy experiments from SPS to LHC, the ratio K^-/K^+ indicates

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the net-strange chemical potential (μ_S), while $\bar{p}/p$ reflects the net-baryon chemical potential (μ_B). The data shows a strong correlation between these ratios, influenced by the net baryon content of the system.

Using the van der Waals interaction, we express $\bar{p}/p$ as:

$$\bar{p}/p = \frac{\bar{p}^{\text{id}}(T,\mu^*)}{1 + bn^{\text{id}}(T,\mu^*)} \Bigg/ \frac{p^{\text{id}}(T,\mu^*)}{1 + b\bar{n}^{\text{id}}(T,\mu^*)} \tag{6}$$

$$\bar{p}/p = \beta(T,\mu^*)\frac{\bar{p}^{\text{id}}}{p^{\text{id}}} \tag{7}$$

where

$$\beta(T,\mu^*) = \frac{1 + bn^{\text{id}}(T,\mu^*)}{1 + b\bar{n}^{\text{id}}(T,\mu^*)} \tag{8}$$

And

$$K^-/K^+ = (K^-)^{\text{id}}/(K^+)^{\text{id}} \tag{9}$$

We analyzed the correlation between K^-/K^+ and $\bar{p}/p$ in nucleus-nucleus collisions across energies from 5 GeV to 2.76 TeV. This correlation is best described by a power-law relationship: [2, 3]

$$K^-/K^+ = (\bar{p}/p)^\alpha \tag{10}$$

where α is a fitting parameter. This correlation provides insights into strangeness production mechanisms and the system's evolution in heavy-ion collisions. In the chemical equilibrium model for ideal HRG, the relationships can be expressed as:

$$K^-/K^+ = \exp((-2\mu_B/3 + 2\mu_S)/T)$$

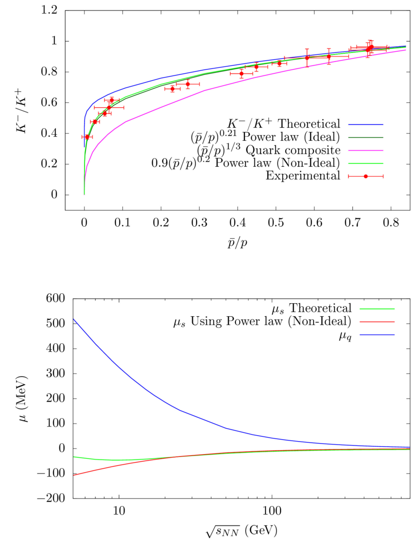
$$\bar{p}^{\text{id}}/p^{\text{id}} = \exp(-2\mu_B/T)$$

where T denotes the chemical freeze-out temperature. Using the fitting procedure, we reduce the expression for μ_s in terms of μ_q:

$$\mu_s = \alpha T \ln(\beta)/2 + \mu_q(1 - 3\alpha) \tag{11}$$

Results and Conclusion

In Fig. (a), we show the experimental data points alongside curves obtained from different approaches. The blue curve represents the van der Waals model calculations, while the magenta curve reflects the light valence quark compositions, yielding $\alpha \approx 0.33$. For comparison, a power-law fit to the theoretical blue curve provides $\alpha \approx 0.18$, contrasting sharply with the light valence quark composition approach. A direct fit to the data using the power law with ideal HRG yields $\alpha \approx 0.21$ (dark green), while for non-ideal HRG, we obtain $\alpha \approx 0.2$ and $\beta \approx 0.98$ (green curve), which closely aligns with our theoretical results [4]. Utilizing this value of α in equation (11) allows us to derive the relation of μ_s as a function of $\sqrt{s_{NN}}$ (T, μ_q) from equation (5), as shown in Fig. (b). The correlation between K^-/K^+ and $\bar{p}/p$ provides critical insights into chemical potentials in ideal and interacting hadron gas models. We identified a power-law correlation with an exponent $\alpha = 0.2$ between K^-/K^+ (net-strange chemical potential, μ_S) and $\bar{p}/p$ (net-baryon chemical potential, μ_B). This relationship, along with the dependence of μ_S and μ_q on $\sqrt{s_{NN}}$, enhances our understanding of hadronic dynamics. Additionally, we derived a simplified equation that describes the dependency of μ_S on μ_q and $\sqrt{s_{NN}}$ for both ideal and non-ideal interacting hadron resonance gas models [3].



a) Correlation between K^-/K^+ and $\bar{p}/p$ ratios, b) Collision Energy Dependence of μ_s and μ_q

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