Gamma spectrum unfolding via diverse algorithms

G. Mishra[1],* Sukanya De[1,2], G. Mohanto[1,2], Ajay Kumar[1], and R.G. Thomas[1,2]
[1]*Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai - 400085, INDIA and*
[2]*Homi Bhabha National Institute, Anushakti Nagar, Mumbai - 400094, INDIA*

Introduction

The unfolding of gamma ray spectrum in general is very crucial for reconstruction of true energy spectrum of gamma-rays originating via different mechanism such as nuclear-reactions or radioactive decay in very different experimental environments. It compensates for the non-ideal response of detectors such as Compton scattering, pair production and limited energy resolution and can help in accurate identification of radionuclides, non destructive assaying of nuclear materials, better environmental monitoring through quantification of low-level contamination etc. This work specifically addresses the unfolding of prompt gamma rays spectrum (PFGS) originating from fission fragments detected in inorganic scintillator detectors. These unfolded PFGS allow deep insight into the understanding of the competition between neutron and gamma emission and the mechanism of excitation energy sorting and angular momentum generation apart from their applied importance in the development of upcoming Gen-IV reactors, Accelerator Driven Systems (ADS) and nuclear safeguards. Here in this work we report different unfolding procedure to extract true gamma ray spectrum from both simulated data as well as experimental data from ^{252}Cf spontaneous fission source.

Experimental & Simulation Details

Measurement of prompt fission gamma rays were carried out using ^{252}Cf spontaneous fission source. Two inorganic CeBr$_3$ (ϕ1.5" x 1.5") detectors were used for gamma spectrum measurements. Each CeBr$_3$ detector was kept at electronic threshold of $\sim 65(\pm 5)$keV. The VME based data acquisition system LAMPS was used to acquire the processed signals data via an ADC and a TDC module. The trigger for the data acquisition system was obtained from the energy loss signal in the ionization chamber produced by the fission fragments. After proper calibration with standard gamma sources and background correction we obtain measured PFGS spectra in both detectors.

We also employ GEANT4 toolkit alongwith FREYA(Fission reaction event yield algorithm) [1] module with DetectorConstruction geometry mimicking the experimental setup and ^{252}Cf as ParticleGun to obtain the simulated PFGS data in CeBr$_3$ detectors. These simulated measured PFGS data will be used to benchmark the unfolding process and to estimate the overall error propagated due to unfolding procedure and statistical noise present in the measured data. To unfold such measured data (experimental and simulated) one also requires to know in detail the response of the detection system at various discrete energy points spanning the energy region of interest of the true spectrum. The GEANT4, in exactly similar detector configuration as employed in experimental run, is used to build the response of system at discrete energy intervals which are later utilized in the unfolding process.

Analysis

Spectrum unfolding is a process mathematically framed as an inverse problem and often represented by the Fredholm integral equation of the first kind:

$$g(y) = \int K(y,x) f(x)\, dx \tag{1}$$

Where, $g(y)$ represents the measured data,

---

*Electronic address: `gmishra@barc.gov.in`

$f(x)$ is the true spectrum we seek to recover, and $K(y,x)$ is the response function, which encapsulates the relationship between the true spectrum and the measured data. In our study, we have utilized iterative unfolding techniques, specifically Gold deconvolution, as detailed in reference [2], along with the GRAVEL unfolding method [3] which has been applied in our previous research work [4]. Additionally, we have also incorporated the rth rank approximation of singular value decomposition(`SVD`) and list square(`LSTSQ`) solver for spectrum unfolding. The `LSTSQ` method employs the LAPACK[6] routine xGELSD behind the scene to find the vector $\mathbf{f}$ that minimizes the Euclidean 2-norm $\|K\mathbf{f} - \mathbf{g}\|_2$. Figure 1 presents a comprehensive comparison of the unfolded experimental results for ^{252}Cf PFGS obtained through various algorithms, juxtaposed with existing literature data [7].

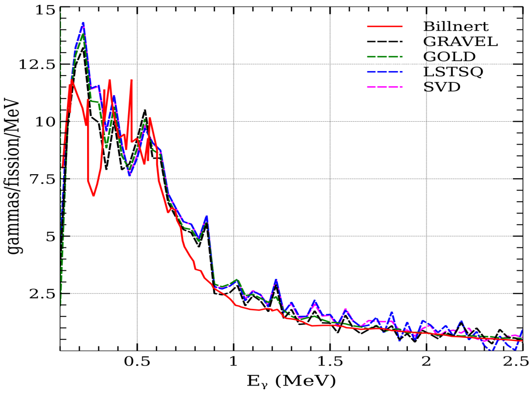

FIG. 1: Unfolding of ^{252}Cf experimental data via different algorithms.

To estimate the error caused by unfolding techniques we use the simulated PFGS data from FREYA based ^{252}Cf source. We simulate the measured data for a finite number of times and unfold the corresponding measured spectrum through all the algorithms. Once the measured spectra are unfolded for a particular algorithm we employ the empirical bootstrapping technique to generate large enough samples in each energy bin and estimate the average statistic and it's spread (within 95% confidence interval). This process is repeated for all the algorithms to get the spread due to unfolding at each energy bin. A comparative plot of unfolded spectrum from different algorithms employed is as shown in fig.2. Although unfolded spectra from different algorithms reasonably agree with each other and overall with the true spectrum, the error(spread) at each energy bin caused inherently due to the unfolding procedure varies from one to another. The unfolded spectra along with error estimates were used to extract the range bound average PFGS characteristics. More details pertaining to this work will be presented later.

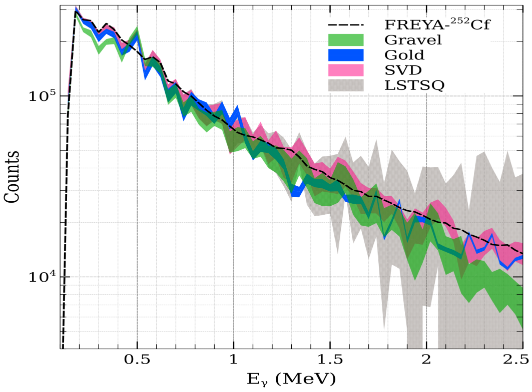

FIG. 2: Unfolded spectrum of simulated ^{252}Cf data and associated uncertainties.

References

[1] J. M. Verbeke et al, Comput. Phys. Commun. 222 (2018) 263.

[2] M. Jandel et al, NIMA 516(1) (2004) 172183.

[3] R. Worrall et al, Fusion Engineering and Design, 161 (2020)112038.

[4] Sukanya De et al, Eur. Phys. J. A, 56(2020) 1-7.

[5] N. Colonna et al, Energy Environ. Sci. 3 (2010) 1910–1917

[6] Anderson, E. et al, LAPACK users' guide. Soc. for ind. and app. math. (1999).

[7] R. Billnert et al, Phys. Rev. C 87 (2013) 024601.