Study of Triaxial deformation variable γ in even – even nuclei

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The deformation parameters β and γ of the collective model [1] are basic description of the nuclear equilibrium shape and structure, while values for these variables have been discussed for many nuclei [2-6]. A systematic study in mass region A=120-140 and A=150-180 [1, 2] can never be the less revealing, we present here such study, in A = 90-120 for Mo, Ru and Pd nuclei where β and γ both vary strongly. We will show the correlations of β and γ with external parameters N_pN_n that may give a simpler description of nuclear structure. In doing this we will use the Davydov and Filippov (DF) model [3]. Even though this model embodies a nuclear shape with rigid triaxiality and these nuclei are known to be γ – soft The expectation of rms values of β and γ extracted from DF model should be valid. Differences between γ – rigid and γ – soft models, mostly show up only in observables which are not used here (such as γ – band energy staggering). In rigid triaxial rotor model (RTRM) the nucleus is considered as rigid rotor with rigid triaxial asymmetry as specified by β and γ. The DF model Hamiltonian can be written as –

$$H = \frac{\hbar^2}{2}\sum_i I_i^2 / J_i \text{.......... ....(1)}$$

Where, I_i are the projection of angular momentum on the intrinsic axes. The moment of inertia are given by the hydrodynamical formula –

$$J_K = \frac{4}{3} J_0 Sin^2(\gamma - \frac{2\pi}{3}K) \text{......... ....(2)}$$

Since J_0 depends on β, J_K involves both deformation parameters. Davydov and Filippov obtain expression for energies and E2 transition probabilities. The $2^+_{1,2}$ states are given by –

$$E2^+_{1,2} = \left(\frac{6\hbar^2}{2J_0}\right)\frac{9-(-1)^{\sigma_{1,2}}\sqrt{81-72\sin^2(3\gamma)}}{4Sin^2(3\gamma)} \text{..(3)}$$

Where, $\sigma_{1,2}$ = 0, 1. The reduced E2 transition probabilities from the $2^+_{1,2}$ states to the ground state can be expressed as –

$$B(E2;2^+_{1,2} \rightarrow 0^+_1) = \frac{e^2Q_0^2}{32\pi}\left[1+(-1)^{\sigma_{1,2}}\frac{3-2X}{\sqrt{9-8X}}\right] \text{.....(4)}$$

Where $Q_0 = 3ZR^2\beta/\sqrt{5\pi}$, $X = \sin^2 3\gamma$ and the value of $B(E2;2^+_2 \rightarrow 2^+_1)$ given by –

$$B(E2;2^+_2 \rightarrow 2^+_1) = \frac{10}{7}\left(\frac{e_2Q_0^2}{16\pi}\right)\left(\frac{X}{9-8X}\right) \text{....(5)}$$

Equation (3), (4) and (5) were used to evaluate the γ deformation parameter of even – even Mo, Ru & Pd nuclei. γ may be obtained by two ways. The ratio $R_2 = E2^+_2/E2^+_1$ depends only on γ. We denote the parameter obtained by these energies by γ_2.

An independent approach is based on the reduced *E2* transition probabilities of the $2^+_{1,2}$

states. Gupta et al have used the experimental quantities $E2_1^+$, $B(E2;2_1^+ \rightarrow 0_1^+)$ and $B(E2;2_2^+ \rightarrow 0_1^+)$ in order to determine β and γ_b but, in many cases the required B (E2) values are unknown. However, often the branching ratio $R_b = B(E2;2_2^+ \rightarrow 0_1^+)/B(E2;2_1^+ \rightarrow 0_1^+)$ is observed which is only a function of γ and allows therefore, determination of this parameter and denoted by γ_b. The asymmetric parameter γ has also been computed using third method viz. from the energy ratio $R_{4/2} = E4_1^+/E2_1^+$ as done by Varshni et al. [5]. The values of γ are listed in table – I along with the values of N_pN_n for Ru, Pd and Mo nuclei. This is clear from the table – I that the product N_pN_n that stands for the proton – neutron interaction strength and is the cause of nuclear deformation in a nucleus, it goes on increasing in ^{92}Mo-^{108}Mo while the established signature of collectivity $R_{4/2} = E4_1^+/E2_1^+$ is not increasing continuously, but it decreases in ^{108}Mo. In case Ru nuclei the maximum value of N_pN_n is 96 in ^{110}Ru for which R_4 is maximum and it decreases to 2.72 for N_pN_n is 84 in ^{112}Ru, while in Pd nuclei it goes increasing continuously while N_pN_n is maximum (=64) in ^{112}Pd and further it goes decreasing 56 to 48 in $^{114-116}$Pd. Such irregular behavior is also present in R_b and R_2 values leaving aside some minor irregularities. The overall variation in Ru – nuclei, R_2 and R_b in the N_pN_n are smooth. Besides, this significant observation in the present study γ derived from R_2 and R_b are almost equal while γ derived from R_4 either not available or their values are large. This reflects internal consistency in asymmetric rotor model relations of energy levels and quadrupole transitions.

This is to be noted that erratic values of γ in some nuclei in mass region A ≈ 150-180 are not present in this A ≈ 90 – 120 mass region.

Table I - The B (E2) ratio R_b, energy ratio $R_{4/2}$, parameter γ along with NpNn are listed for Ru, Pd and Mo – nuclei.

Nucleus	N_pN_n	R_b (γ_b in deg.)	R_4 (γ_4 in degree)	R_2 γ_2(in degree)
^{92}Mo	0	-	1.51 (>30)	2.05 (28)
^{94}Mo	16	-	1.81 (>30)	2.14 (27)
^{96}Mo	32	-	2.09 (>30)	2.09 (28)
^{98}Mo	48	-	1.73 (>30)	2.28 (30)
^{100}Mo	64	30.3 (25.9)	2.12 (>30)	1.99 (30)
^{102}Mo	80	-	2.51 (>30)	2.86 (23)
^{104}Mo	96	5.8 (20.2)	2.92 (23)	4.22 (19)
^{106}Mo	112	4.2 (17.5)	3.04 (21)	4.14 (19)
^{108}Mo	128	13.7 (23.3)	2.92 (23)	3.04 (22)
^{98}Ru	24	50.0 (26.8)	2.14 (>30)	2.17 (27)
^{100}Ru	36	24.4 (25.5)	2.27 (>30)	2.52 (24)
^{102}Ru	48	26.3 (25.7)	2.33 (>30)	2.32(25.5)
^{104}Ru	60	18.52	2.48 (>30)	2.49(24.5)
^{106}Ru	72	(24.5)	2.65 (30)	2.93(22.5)
^{108}Ru	84	-	2.75 (27)	2.93(22.5)
^{110}Ru	96	10.2 (22.3)	2.75 (27)	2.54 (24)
^{112}Ru	84	14.2 (23.8)	2.72 (27.5)	2.21 (26)
^{102}Pd	24	25.3 (25.5)	2.29 (>30)	2.76 (23)
^{104}Pd	32	8.3 (21.5)	2.37 (>30)	2.41 (25)
^{106}Pd	40	20.0 (25.0)	2.40 (>30)	2.20(26.5)
^{108}Pd	48	37.4 (26.3)	2.41 (>30)	2.15 (27)
^{110}Pd	56	71.4 (27.3)	2.46 (>30)	2.18(26.7)
^{112}Pd	64	71.4 (27.3)	2.54 (>30)	2.11 (28)
^{114}Pd	56	-	2.56 (>30)	2.09 (28)
^{116}Pd	48	-	2.58 (>30)	2.16 (27)

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