Neutrinoless double beta decay of ^{76}Ge in the Deformed Shell Model

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Introduction

Neutrinoless double beta decay ($0\nu\beta^-\beta^-$) is a rare weak interaction process which can occur only if neutrino is a Majorana particle and the conservation of lepton number is voilated. Recent experimental measurements suggest that neutrinos have mass. Observation of $0\nu\beta^-\beta^-$ can give absolute neutrino mass provided the nuclear matrix elements are known. In view of the above, experimental programmes have been initiated at different laboratories across the globe (including the INO project in India) to observe this decay. Except for a claim by Heidelberg-Moscow group for ^{76}Ge [1], this process has not yet been observed experimentally. On the other hand two neutrino double beta decay has been expeimentally observed for more than 20 nuclei. Hence this process provides critical test of the goodness of the nuclear models.

Deformed shell model (DSM) based on Hartree-Fock states with band mixing has been established to be very successful in describing various spectroscopic properties [band structures, shapes, band crossings, $B(E2)$'s and so on] of nuclei in the A=60-90 mass region (also in A=44-60); see [2] for details. More importantly, this model has also been used for studying 2ν double beta decay, in a first attempt, for ^{76} Ge $\rightarrow$ ^{76}Se in [3] with considerable success. More recently, in [4] we have applied DSM to study β-decay half lives, GT distributions, electron capture rates and 2ν e^+DBD in ^{78}Kr, 2ν e^+DBD in ^{74}Se [5] and ^{84}Sr [6] and 2ν DBD in ^{82}Se [7].

The success of the model in describing the two-neutrino double beta decay has encouraged us to apply the model to study the $0\nu\beta^-\beta^-$.

Formalism

Half-life for $0\nu\beta^-\beta^-$ for the 0_i^+ ground state (gs) of a initial even-even nucleus decay to the 0_f^+ gs of the final even-even nucleus, with a few approximations [8], is given by

$$
\begin{aligned}
&[T_{1/2}^{0\nu}(0_i^+ \rightarrow 0_f^+)]^{-1} = G^{0\nu}|M^{0\nu}|^2 \frac{\langle m_\nu \rangle^2}{m_e^2} \;; \\
&M^{0\nu} = M_{GT}^{0\nu} - \frac{g_V^2}{g_A^2} M_F^{0\nu} \\
&\qquad = \left\langle 0_f^+ \mid\mid \mathcal{O}(2:0\nu) \mid\mid 0_i^+ \right\rangle \;, \\
&\mathcal{O}(2:0\nu) = \sum_{a,b} \mathcal{H}(r_{ab}, \overline{E}) \tau_a^+ \tau_b^+ \left(\sigma_a \cdot \sigma_b - \frac{g_V^2}{g_A^2} \right), \\
&\mathcal{H}(r_{ab}, \overline{E}) \rightarrow \mathcal{H}_{eff}(r_{ab}, \overline{E}) \\
&\qquad = \frac{R}{r_{ab}} \Phi(r_{ab}, \overline{E}) \\
&\qquad \times \left[1 - \exp(-\gamma_1 r_{ab}^2) \left(1 - \gamma_2 r_{ab}^2\right) \right]^2 \;.
\end{aligned} \tag{1}
$$

In the above

$$
\Phi(r_{ab}, \overline{E}) = \frac{2}{\pi} \left[\sin\left(\frac{\overline{E}\, r_{ab}}{\hbar c}\right) f_1\left(\frac{\overline{E}\, r_{ab}}{\hbar c}\right) - \cos\left(\frac{\overline{E}\, r_{ab}}{\hbar c}\right) f_2\left(\frac{\overline{E}\, r_{ab}}{\hbar c}\right)\right] . \tag{2}
$$

In Eq. (2), $f_1(x) = -\int_x^\infty t^{-1}\cos t dt = Ci(x) = \gamma + \ln x + \int_0^x t^{-1}(\cos t - 1)dt$ and $f_2(x) = -\int_x^\infty t^{-1}\sin t dt = Si(x) - \frac{\pi}{2}$; $Si(x)$ and $Ci(x)$ are the sine and cosine integrals. In (1), $M^{0\nu}$ is NTME and $\langle m_\nu \rangle$ is the average neutrino mass. The $G^{0\nu}$ are phase space integrals and tabulations for them are available. The g_A and g_V are the weak axial-vector and vector coupling constants (we use g_A/g_V=1).

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The $\mathcal{H}_{eff}(r_{ab}, \overline{E})$ is the 'neutrino potential' with short range correlations incorporated. The parameters $\gamma_1 = 1.1$ fm^{-2}, $\gamma_2 = 0.68$ fm^{-2}, $R = 1.2A^{1/3}$ fm, $\overline{E} = 1.12A^{1/2}$ MeV, r_{ab} in fm and $\hbar c = 197.327$ MeV fm. Here we also need the oscillator length parameter $b = \sqrt{\hbar/m\omega} \sim 0.9A^{1/6}$ fm. Calculation of the two-body matrix elements of the 0ν transition operator $\mathcal{O}(2:0\nu)$ involves Talmi integrals and Brody-Moshnisky brackets. The antisymmetrized form of the TBME is given by

$$
\begin{aligned}
&\langle (j_1^p\, j_2^p) JM \mid \mathcal{O}(2:0\nu) \mid (j_3^n\, j_4^n) JM\rangle_a = \\
&\frac{1}{\sqrt{(1+\delta_{j_1^p j_2^p})(1+\delta_{j_3^n j_4^n})}} \times \sum_{L,S} \\
&\left[2S(S+1) - 3 - \frac{g_V^2}{g_A^2}\right] \chi \begin{Bmatrix} \ell_1^p & \ell_2^p & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1^p & j_2^p & J \end{Bmatrix} \\
&\chi \begin{Bmatrix} \ell_3^n & \ell_4^n & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_3^n & j_4^n & J \end{Bmatrix} \times \sum_{n,\ell;N,L'} \left[1 + (-1)^{\ell+S}\right] \\
&\langle n\ell, NL', L \mid \mathbf{n}_1^p \ell_1^p, \mathbf{n}_2^p \ell_2^p, L\rangle \\
&\times \langle n'\ell, NL', L \mid \mathbf{n}_3^n \ell_3^n, \mathbf{n}_4^n \ell_4^n, L\rangle \\
&\times \sum_p B(n\ell, n'\ell, p)\, I_p\ .
\end{aligned}
\tag{3}
$$

In the above, I_p is the Talmi integral. The coefficients $B(n\ell, n'\ell, p)$ are given by Brody, Jacob and Moshinsky in [9].

Results and Discussion

Calculations of NTME using DSM in $f_{5/2}pg_{9/2}$ space with a modified Kuo interaction have been carried out for ^{76}Ge. It is found that for calculating the energy spectra and the electro-magnetic properties, a few lowlying configurations are found to be sufficient. However, for calculating GT properties, one has to consider sufficiently large number of configurations. For example, taking about 20 configurations, the NTME $M^{0\nu}$ is only about 1. However, making a large calculation with 287 configurations for ^{76}Ge and 201 configurations for ^{76}Se for generating the ground states, the NTME is found to rise to 5.22. This value is close to QRPA values. All calculational details will be presented in the meeting. We are now in the process of performing such large calculations for ^{82}Se and similarly for 0ν e^+DBD for ^{74}Se and ^{84}Sr nuclei.

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