On Absorption of Fourth Order Wigner-Kirkwood Energy into the Second Order Term

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Introduction

The semi - classical Wigner Kirkwood (WK) expansion method [1–3] has recently been applied successfully to calculate shell corrections for spherical nuclei [4]. It has been shown that these shell corrections, along with a simple six parameter liquid drop formula, yields good description of ground state masses of spherical nuclei spanning the entire periodic table [4]. The model has also been applied to calculate the binding energies of few deformed nuclei, with a good degree of success [5, 6].

It has been observed [4] that the Wigner - Kirkwood expansion is required to be carried out upto the fourth order in $\hbar$ to achieve sufficient degree of accuracy in the shell corrections. The fourth order calculations, however are quite complex, and it is highly desirable to check if the fourth order effects can be absorbed into the second order correction terms. Recently, it has been demonstrated [7] that such absorption is indeed possible, and the shell corrections obtained by using the resulting second order expansion is within ~ 100 KeV of the shell corrections obtained by using the full WK expansion upto fourth order. Here, we report a quantitative investigation of the *effective* second order WK expansion with respect to the shell corrections and hence the determination of ground state masses of the nuclei spanning the entire periodic table.

Results and Discussion

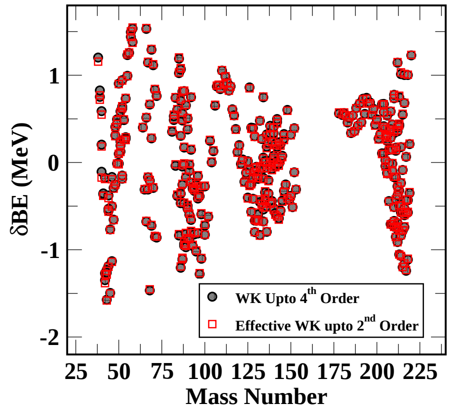

FIG. 1: Difference between the calculated and the experimental nuclear masses. See text for details.

To quantify the degree of closeness of the *effective* WK expansion to the full WK expansion, we carry out two sets of calculations: 1) The shell corrections are obtained for a set of 367 spherical nuclei [4] with the full WK

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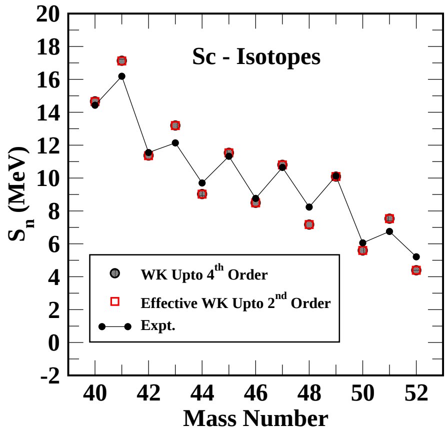

FIG. 2: The calculated and the experimental single neutron separation energies for Sc isotopes.

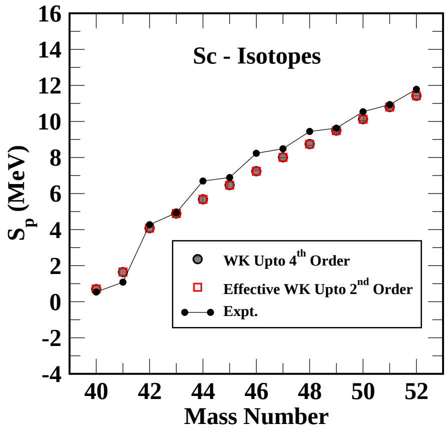

FIG. 3: The calculated and the experimental single proton separation energies for Sc isotopes.

expansion. These, along with the pairing energies within the Lipkin - Nogami scheme constitute the 'micro' part of the nuclear masses. The liquid drop parameters are then determined by a χ^2 fit to the experimental masses [8]. The fitted masses have *rms* deviation of 630 KeV. 2) The step 1 is repeated with effective WK expansion, and it is found that the liquid drop parameters thus obtained are practically the same as before, and so is the *rms* deviation in masses, thereby reflecting the reliability of the effective WK expansion to the second order in $\hbar$, modified to absorb the fourth order effects.

As an illustration, three plots have been presented hereunder. The differences between the calculated (within both the WK expansions) and the experimental nuclear masses for the 367 nuclei considered here have been depicted in Fig. (1). It can be observed that the two approaches yield practically the same degree of agreement with the experimental data. Further, the single neutron and proton separation energies for Sc isotopes have been plotted in Figs. (2) and (3). Reliability of the present approach is amply clear from these figures. It is interesting to note from Fig. (3) that the proton separation energy changes quite substantially with changing neutron number for a given chain of isotopes.

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