Energy ratios from power index formula

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For well deformed (even Z even N) nuclei, the ideal rotor formula $E_I=AI(I+1)$ provides a good approximation for the level energies [1]. But for the shape transitional nuclei this formula or its extensions to two or three terms are inadequate. Various energy formulae, such as the soft rotor formula (SRF), anharmonic vibrator (AHV), Rotation-Vibration Interaction (RVI) formula and Variable Moment of Inertia (VMI) model etc. are in use. All of these are of the form of an expansion in powers of spin I, or its combinations, limited to two or three terms. However, Gupta et al. [2] proposed a single term formula in the form of a power index expression (1):

$$E=a\ I^b. \qquad (1)$$

Here, I is the spin angular momentum, and a, b are the parameters. It was noted in [2] that the power index 'b' and the energy scaler coefficient 'a' are fairly constant for varying spin for I <12 for any given nucleus. This was verified for the rare earth nuclei in the A-150-200 region [2]. Mittal et al. [3] verified the validity of this for the light N<82 Xe-Gd nuclei. For example, in $^{148\text{-}152}$Nd isotopes, ($R_{4/2}$ =2.50 to 3.27) average b' are 1.311, 1.557, 1.732 respectively with ~1% variation upto 12^+ and 'a' are 121.5, 44.07, 21.75 respectively with 2-3 % variation upto 12^+ in each isotope (see [4]).

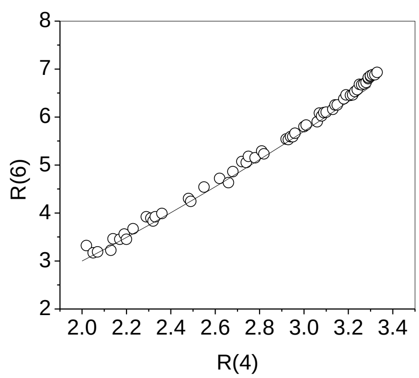

Fig. 1. R(6) vs R(4) in $E=aI^b$ (curve), Ex data (points).

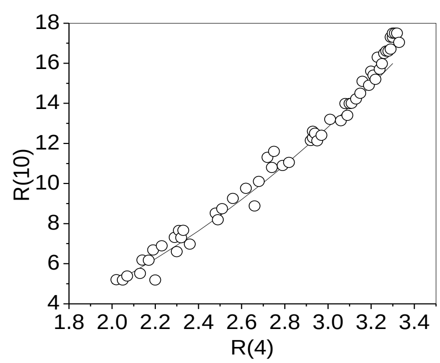

Fig. 2. R(10) vs R(4) in $E=aI^b$ (curve), Ex data (points).

Here we study how good are the energy ratios $R_I(=E_I/E_2)$ from Eq, (1) with respect to the ratio R(4), for I=6, 8, 10, and compare it with the two parameter SRF formula (Eq.2) [5].

$$E(I)=A\ I(I+1)/(1+\sigma I). \qquad (2)$$

This is more transparently seen on the Mallmann plot for energy ratios [6].

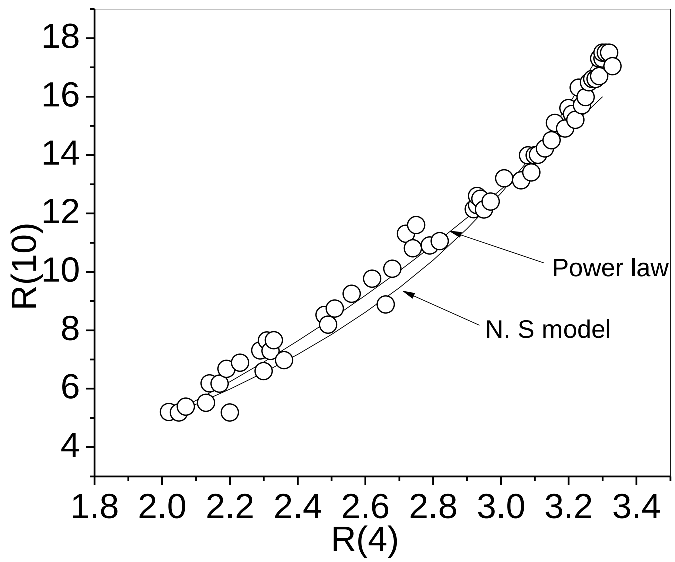

Fig. 3. The energy ratio R(10) versus R(4).

Results

Fig. 1 is the plot of the energy ratio R(6)(=E_6/E_2) versus R(4). The continuous curve represents the values from the power index formula Eq. (1). For all R_I of a nucleus, same average 'b' is used. The circles are experimental data. Fig. 2 depicts the same data for R(10). There is good agreement in both cases for the whole range. The theory curve is slightly on the under side. This fact is on account of the slight effect of the RVI (centrifugal stretching) on the nuclear core, which increases with spin. For well deformed nuclei, the deviations are larger. This is on account of the use of spin I in Eq. (1) for the nuclei which follows the I(I+1) dependence.

Fig. 3 depicts also the values from the SRF formula (Eq. 2) [5]. The deviation from experiment is relatively larger, especially in the central portion. This signifies the better representation of level energies of the ground band of shape transitional nuclei by the power index formula, wherein the condition of integral powers of I is relaxed, and non-integer index (between 1 and 2) is allowed. It signifies the advantage in relaxing the rotational model limitation by a generalized expression, independent of rotor model for shape transitional nuclei.

References

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