The $^3P_2$ and $^3F_2$ n-n scattering phase shifts using Reid soft core potential

Arun K. Jain, Sudhir R. Jain, and B. N. Joshi
Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai - 400 085

It is a tough job to evaluate the $^3P_2$ and $^3F_2$ scattering phase shifts for the neutron-neutron (n-n) system due to the presence of the tensor interaction coupling the two ℓ-values(ℓ= 1 and 3). As the orbital angular momentum ℓ is not a constant of motion in the presence of tensor interaction the triplet states with the same angular momentum J have two orbital angular momenta (1 and 3) coupled. Normally one resorts to large matrix inversion techniques to numerically evaluate the phase shifts at various energies. In this process one loses the physical insight about the impact of the tensor interaction such as its amount, position and nature of the coupling. A semi-classical method developed for the deuteron and the $^3S_1$ and $^3D_1$ n-p scattering phase shifts has been extended here to decouple equations even in the presence of the tensor interaction for the $^3P_2$ and $^3F_2$ n-n scattering. First of all the tensor operator S_{12} operating on the $^3P_2$ and $^3F_2$ spin-angle wave functions Y^M_{JLS} are evaluated and we get

$$Y^2_{211}u(r) = Y^1_1(\theta,\phi)\chi^1_1 u(r)$$

$$Y^2_{231}w(r) = (\frac{1}{21})^{\frac{1}{2}}(Y^1_3(\theta,\phi)\chi^1_1 - \sqrt{5}Y^2_3(\theta,\phi)\chi^0_1 + \sqrt{15}Y^3_3(\theta,\phi)\chi^0_1)w(r).$$

We now get

$$S_{12}Y^2_{211} = \frac{6\sqrt{6}}{5}Y^2_{231} - \frac{2}{5}Y^2_{211}$$

$$S_{12}Y^2_{231} = \frac{6\sqrt{6}}{5}Y^2_{211} + \frac{8}{5}Y^2_{231}.$$

We now use the Reid soft core potential for $T = 1$ n-n scattering for ($^3P_2$ and $^3F_2$) as follows

$$V(r) = V_c + V_t S_{12} + V_{LS}(\mathbf{L}.\mathbf{S}) \qquad (1)$$

with

$$V_c(r) = -10.463/3e_1(x) - 933.48e_4(x) - 4152.1e_6(x) + 4152.1e_6(x), \qquad (2)$$

$$V_t(r) = 10.463 \left[\left(\frac{1}{3}+\frac{1}{x}+\frac{1}{x^2}\right)e_1(x) - \left(\frac{4}{x}+\frac{1}{x^2}\right)e_4(x)\right] -34.925e_3(x), \qquad (3)$$

$$V_{LS} = -2074.1e_6(x)$$

where $e_n(x) = e^{-nx}/x$ and $x = 0.7r$. Writing in a matrix eigenvalue form,

$$\mathbf{H}\Psi = E\Psi \qquad (4)$$

with

$$\Psi = \begin{bmatrix} u \\ w \end{bmatrix}$$

$$\mathbf{H} = \left(\frac{-\hbar^2}{M}\frac{d^2}{dr^2}\right)\mathbf{I} + \mathbf{V}(r),$$

$$\mathbf{V}(r) = \begin{bmatrix} v_{11} & \frac{6\sqrt{6}}{5}V_t(r) \\ \frac{6\sqrt{6}}{5}V_t(r) & v_{22} \end{bmatrix}.$$

Here, $\mathbf{I}$ is an identity matrix. $v_{11} = V_c + 3V_{LS} - \frac{2}{5}V_t + \frac{e^2}{r} + \frac{2\hbar^2}{Mr^2}$ and $v_{22} = V_c - 4V_{LS} + \frac{8}{5}V_t + \frac{e^2}{r} + \frac{12\hbar^2}{Mr^2}$. Potential matrix $\mathbf{V}$ being real-symmetric, the diagonalizing transformation is orthogonal $\mathbf{O}$, such that

$$\mathbf{O}(r)^T\mathbf{V}(r)\mathbf{O}(r) = \mathbf{v}(r) = \begin{bmatrix} v_+(r) & 0 \\ 0 & v_-(r) \end{bmatrix},$$

$$\text{where } \mathbf{O}(r) = \begin{bmatrix} \cos\theta/2 & \sin\theta/2 \\ -\sin\theta/2 & \cos\theta/2 \end{bmatrix},$$

$$\text{with } \tan\theta = \frac{12\sqrt{6}V_t(r)}{\frac{50\hbar^2}{Mr^2} + 10V_t(r) - 35V_{LS}(r)}$$

Since $-i\hbar\frac{d}{dr}(= p_r)$ does not commute with r, there appears a vector potential $\mathbf{A}$ but we note that it is purely off-diagonal and carries an order of $\hbar$ with it. We take only terms of order $O(\hbar^0)$ and take the similarity-transformed Hamiltonian, $\mathbf{H}^{(1)} = \mathbf{O}^T\mathbf{H}\mathbf{O}$ to get diagonalized form of the Hamiltonian as $\tilde{\mathbf{H}}_{\mu\nu} = h^\mu\delta_{\mu\nu}$ where

$$h^{(\pm)} = \left(-\frac{\hbar^2}{M}\right)\left[1 \pm \frac{\hbar^2}{M(v_+(r) - v_-(r))}\left(\frac{d\theta(r)}{dr}\right)^2\right] * \frac{d^2}{dr^2} + v_\pm(r) + \frac{\hbar^2}{8M}\left(\frac{d\theta(r)}{dr}\right)^2. \quad (5)$$

With this,

$$\psi^+(r) = u(r)\cos\frac{\theta(r)}{2} - w(r)\sin\frac{\theta(r)}{2},$$

$$\psi^-(r) = u(r)\sin\frac{\theta(r)}{2} + w(r)\cos\frac{\theta(r)}{2}. \quad (6)$$

For large values of r, we get $\theta \sim 0$. Therefore, as $r \to \infty$,

$$\psi^+(r) = u(r), \quad \psi^-(r) = w(r).$$

These equations are numerically solved using the Runga-Kutta method and matching $\psi(r)$ and their derivatives with the asymptotic scattering state wave functions the phase shifts are evaluated. Thus one obtains the phase shifts for $^3P_2$ and $^3F_2$ n-n scattering states.

In Fig. 1, the results are in good agreement with large matrix diagonalizations results[3] as

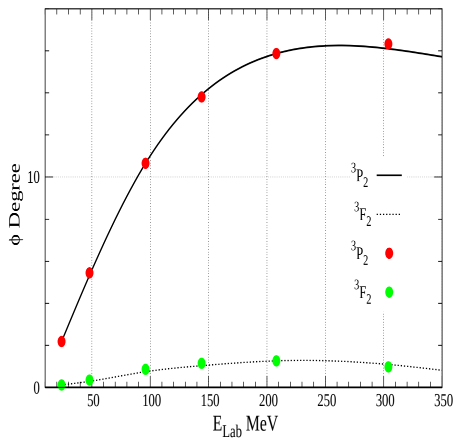

FIG. 1: $^3P_2$ and $^3F_2$ n-n scattering Phase shifts as a function of lab energy

well as with the experimental values obtained by Arndt and MacGregor[4]. Good agreement indicates the success of our semi-classical wave equations which are quite simple and physically appealing. The wave functions $u(r)$ and $w(r)$ can be used to evaluate the $t(r)$ effective interactions.

References

[1] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).
[2] S. R. Jain, J. Phys. G **30**, 157 (2004).
[3] R. V. Reid. Ann. of Phys. **50**, 411 (1968).
[4] R. A. Arndt and M.H. MacGregor, Phys. Rev. **141**, 873 (1966).