Quark Matter Viscosity

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Introduction

A lot of interest in the study of viscosity of quark matter has been stimulated in the recent years by the experiments carried out at the relativistic heavy-ion collider (RHIC), which seem to indicate that above the critical temperature the quark-gluon plasma (QGP) formed in these experiments behaves as a fluid – a 'strongly-coupled matter', with quite a small viscosity (i.e. almost a perfect liquid). This conclusion is mostly based on the fact that QGP formed in RHIC was well described by an ideal hydrodynamics model, and the findings on the v_2 coefficient measured in the multipole analysis of the angular distribution of the hadrons produced in the ultrarelativistic collisions suggesting substantial collective flow ("elliptic" flow) to imply that the viscosity cannot be very large. Thus, transport properties of quark mater are of great interest both experimentally and theoretically.

It is also known that viscosity of nuclear matter plays an important role in the damping of neutron star oscillations and instabilities. Information about quark matter viscosity is essential in understanding damping of non-radial modes of oscillations in hybrid stars that is neutron stars with quark matter in the core. In this note, we present preliminary results of the calculations of quark matter viscosity in the framework of Nambu Jonalasinio model.

Formulation

Transport coefficients like the thermal conductivity and the shear and bulk viscosities of a system control the deviations of equilibrium expectation value of the energy momentum tensor caused by the presence of gradients in thermodynamic parameters like the temperature and the four-velocity. The energy momentum tensor, $T^{\mu\nu}$, can be decomposed in to different physical processes as,

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + P^\mu u^\nu + P^\nu u^\mu + \pi^{\mu\nu} \,,$$

with $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, and, ϵ, p, P^μ and $\pi^{\mu\nu}$ are respectively the energy density, pressure, heat current and viscous shear stress. The metric tensor is $g^{\mu\nu}$, and the four-velocity vector, $u^\mu(x)$, satisfies, $u^\mu(x)u_\mu(x) = 1$.

Expanding, $\pi^{\mu\nu}$, in gradients of four-velocity vector,

$$\delta \langle \pi^{\mu\nu} \rangle = \eta \biggl(\partial^\mu u^\nu + \partial^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\Delta^{\rho\sigma}\partial^\rho u^\sigma \biggr) + \cdots \,,$$

we identify, η, the co-efficient of the term linear in gradient of four-velocity vector as the shear viscosity.

There are two basic approaches to calculate transport coefficients, (1) transport theory and (2) response theory. In the framework of response theory, the Kubo formula gives the shear viscosity $\eta(\omega)$ at temperature T as,

$$\eta(\omega) = \frac{1}{T}\int_0^\infty dt\, e^{i\omega t} \int d\mathbf{r}\, (J_{xy}(\mathbf{r},t), J_{xy}(0,0)) \quad (1)$$

where J_{xy} is the x, y component of the energy-momentum tensor of the quark matter with the correlation function in the right-hand side defined as,

$$(A, B) \equiv \beta^{-1}\int_0^\beta d\lambda \left\langle e^{\lambda H} A e^{-\lambda H} B \right\rangle$$

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where A and B are operators of any physical quantity and H denotes our Hamiltonian. $\langle Q \rangle = Tr\left(Qe^{-\beta H}\right)/Tr\left(e^{-\beta H}\right)$ means the thermal average at temperature $T(\beta = 1/T)$.

The Nambu JonaLasinio (NJL) model [1] is a phenomenological quark model, which takes into account the chiral symmetry breaking at low density and temperature, and chiral symmetry restoration at high density and temperature. In the broken chiral symmetry phase, quarks develop a dynamical mass by their interaction with the vacuum.

The shear viscosity, $\eta = \eta(\omega = 0)$, of quark matter in the NJL model can be obtained from (1) following [2, 3] as,

$$\eta = \frac{64 N_c N_f}{T} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int \frac{d^3p}{(2\pi)^3} \frac{p_x^2 p_y^2 M^2 \Gamma^2 n(\epsilon)\left(1 - n(\epsilon)\right)}{\left[\left(p^2 - M^2 + \Gamma^2\right)^2 + 4M^2\Gamma^2\right]^2}, \quad (2)$$

where, $p^2 = \epsilon^2 - \vec{p}^2$, and N_C and N_f are the colour and flavour numbers respectively. The Fermi distribution function, $n(\epsilon)$, also gives additional dependence of temperature and density (chemical potential) that make η density and temperature dependent.

The quark mass, M, calculated in the 3-flavour NJL model is density dependent, and the quasi-particle width, Γ, following [2] we assume,

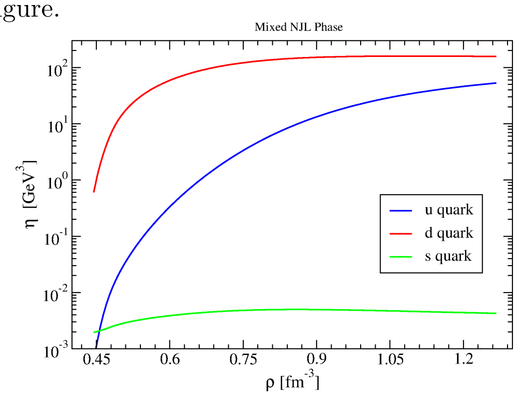
$$\Gamma(p) = \frac{\lambda^2 M^2}{\sqrt{|\vec{p}|^2 + M^2}}$$

in which λ is a dimensionless constant parameter reflecting the strength of the interaction [1],

$$\lambda^2 = \left(G_S + G_D \langle \bar{s}s \rangle\right) \Lambda^2 \,,$$

where G_S and G_D are four point and six point coupling constants of NJL model, $\langle \bar{s}s \rangle$ is the strange quark condensate and Λ is the cut-off parameter.

We compute η for beta equilibrated quark matter from (2) taking parameters of NJL model from [4]. The results are shown in the figure.



In figure ρ represents baryon density. The shear viscosity of the quark matter is known taking density average

We plan to compute shear viscosity of nuclear matter also using this formalism, and determine the damping time scale of non-radial oscillations of hybrid stars.

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