Semileptonic decay of $\Lambda_b \to X_c l \nu_l$ in quark-diquark model

Ajay Majethiya[1,3],* Kaushal Thakkar[2,3], and P C Vinodkumar[3]
[1]Kalol Institute of Technology and Research centre, Kalol
[2]Applied Physics Department, S. V. National Institute of Technology, Surat-395 007,INDIA and
[3]Department of Physics Sardar Patel University,Vallabh Vidyanagar, INDIA

Introduction

Among all the processes, the semileptonic decay of hadrons plays an important role for probing the success and predictability of the model that describes the hadron state. This decay process is relatively simple and less dependent on the non-perturbative QCD effects as the leptons do not participate in strong interaction. And there is no contamination from the crossed gluon-exchanges between quarks residing in different hadrons which are produced in the weak transitions. Thus one might gain more model-independent information, such as extraction of the Cabibbo-Kobayashi-Maskawa matrix elements from semileptonic decay rates. In the semi-leptonic decays of heavy hadrons, it is expected to factorize the perturbative and non-perturbative parts more naturally. Recently, semileptonic decay of Λ_b has been reported in the light-front quark model [1]. Here, we employ the quark-diquark model to study the semileptonic decay of $\Lambda_b \to X_c l \nu_l$. The confinement potential is assumed as two body colour coulomb plus power potential with exponent ν.

Methodology

The Hamiltonian of the baryon, in the quark-diquark model, can be written in terms of diquark Hamiltonian plus quark-diquark Hamiltonian as [2]

$$H = H_{jk} + H_{i,jk} \quad (1)$$

The internal motion of the diquark(jk) is described by

$$H_d = H_{jk} = \frac{p^2}{2m_{jk}} + V_{jk}(r_{jk}) \quad (2)$$

where, p is the relative momentum of the quarks within the diquark. The Hamiltonian of the relative motion of the diquark(jk) and the third quark(i) is given by

$$H_{i,d} = H_{i,jk} = \frac{q^2}{2m_{i,jk}} + V_{i,jk}(r_{id}) \quad (3)$$

where, q is the relative momentum between the diquark and the third quark, $m_{jk} = \frac{m_j m_k}{m_j+m_k}$ and $m_{i,jk} = \frac{m_i(m_j+m_k)}{m_1+m_2+m_3}$.
In the diquark model, the potential energy can be written, as

$$V = V_{jk}(r_{jk}) + V_{i,jk}(r_{id}) \quad (4)$$

Where, the diquark potential as well as the quark-diquark potential are assumed to be, [3]

$$V_{jk} = -\frac{2}{3}\alpha_s \frac{1}{r_{jk}} + b\, r_{jk}^{\nu}; V_{i,jk} = -\frac{4}{3}\alpha_s \frac{1}{r_{id}} + b\, r_{id}^{\nu} \quad (5)$$

respectively. Here, ν is the potential exponent, r_{id} is the quark-diquark separation distance, α_s the running strong coupling constant, b is the model parameter corresponding to the confining part of the potential, which is assumed to be same for the di-quark interaction as well as between the quark-diquark interaction. The numerical approach using the Runge-Kutta method in a mathematica note book has been used to solve the Schrodinger equation corresponds to the Hamiltonian as given in Eq.2 and 3. The degeneracy of the states are removed by introducing the spin dependent interaction potential given by [3].

Semileptonic decay of $\Lambda_b \to X_c l \nu_l$

The inclusive semileptonic decay width of the beauty baryon $\Lambda_b \to X_c l \nu_l$, where, X_c represents sum over all charmed hadrons, is given by [4, 5]

*Electronic address: ajay.phy@gmail.com

$$\Gamma^{inc} = \Gamma_0(1 - \frac{2}{3}\frac{\alpha_s}{\pi}g(x)) \quad (6)$$

Where, $x = (m_c/m_b)^2$ and Γ_0 is the lowest order free quark decay and is expressed as [4]

$$\Gamma_0 = \frac{G_F^2|V_{cb}|^2 m_b^5}{192\pi^3} I_0(x) \quad (7)$$

the function $g(x)$ is written as [4, 5]

$$g(x) = h(x)/I_0(x) \; ,$$

Where,

$$\begin{aligned} h(x) &= -(1-x^2)\left(\frac{25}{4} - \frac{239}{3}x + \frac{25}{4}x^2\right) \\ &+x\ln(x)\left(20 + 90x - \frac{4}{3}x^2 + \frac{17}{3}x^3\right) \\ &+x^2\ln^2(x)(36 + x^2) + \\ &(1-x^2)\left(\frac{17}{3} - \frac{64}{3} + \frac{17}{3}x^2\right)\ln(1-x) \\ &-4(1 + 30x^2 + x^4)\ln(x)\ln(1-x) \\ &-(1 + 16x^2 + x^4)[6\mathrm{Li}_2(x) - \pi^2] \\ &-32x^{3/2}(1+x)\left[\pi^2 - \right. \\ &4\mathrm{Li}_2(\sqrt{x}) + 4\mathrm{Li}_2(-\sqrt{x}) \\ &\left. -2\ln(x)\ln\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\right] \end{aligned} \quad (8)$$

$$I_0(x) = (1-x^2)(1-8x+x^2) - 12x^2\ln x$$

Here, $Li_2(x) = \frac{x}{1} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + ...$
As the decay width here depends on the mass of the heavy quarks within the initial (Λ_b) baryon, the non perturbative bound state effects on the decaying b-quark within Λ_b is considered in our calculations by accounting an effective mass to the bound quarks (ecqm). We compute the inclusive decay width with and without considering the bound state effect on the quark mass parameter. The semileptonic decay widths thus computed here are listed in Table I and are compared with other model predictions.

TABLE I: Semileptonic decay widths (Γ_{SL}) of beauty baryons (* in $10^{10}s^{-1}$.) $m_c = 1.5$ GeV

Decay	ν	m_b	M_{Λ_b}	Γ_{SL}		Others
$\Lambda_b \to X_c l \nu_l$				cqm	ecqm	
	0.4	4.450	5.660	2.50	6.09	
	0.6	4.550	5.657	2.97	6.22	6.50 [4]
Set A	0.8	4.600	5.655	3.24	6.28	
(α_s=0.15)	1.0	4.630	5.654	3.40	6.31	
	0.2	4.570	5.656	3.08	6.24	
	0.4	4.720	5.651	3.93	6.42	
Set B	0.6	4.780	5.647	4.32	6.49	
(α_s=0.20)	0.8	4.815	5.647	4.55	6.54	
	1.0	4.840	5.647	4.73	6.57	

Results and discussion

The semileptonic decay computed here with set of parameters (A and B) are listed with the other theoretical results. Our results with the parametric set B, with considering the effective quark mass (cqm) of the semileptonic decay width for $\Lambda_b \to X_c l \nu_l$ lie in the narrow range of (6.24-6.57) $*10^{10}s^{-1}$ and are in good agreement with the theoretical results reported by [4]. We find that the present results are not much sensitive to the choices of the interquark potential, however, the bound state effects (with ecqm) are found to be important in the semileptonic decay of $\Lambda_b \to X_c l \nu_l$.

References

[1] Hong-Wei Ke et. al, Phys. Rev. D **77**, 014020 (2008).

[2] W. S. Carvalho *et al.*, arXiv:hep-ph **9404298v1**,(1994).

[3] S. S. Gershtein *et al.*, arXiv:hep-ph **9811212v2**,(2000).

[4] J. G. Korner et. al, Phys. Rev. D **62**, 074008 (2000).

[5] Y. Nir, Phys. Lett. B **221**, 184 (1989).