Multifractality in the emission spectra of projectile fragments in ^{24}Mg-Em interactions at 4.5 AGeV

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Introduction

The self similarity observed in the power law dependence of scaled factorial moments reveals a connection between intermittency and fractality. In multifractal approach, it has been suggested that the nuclear interactions can be treated as geometrical objects with non-integer dimensions. The study of chaotic system and intermittent behavior in turbulent fluids has been performed using the fractal dimension. Out of various methods that have been proposed to investigate the fractal structure, Hwa[1] was the first to provide the idea of the multifractal moments G_q to study the multifractality and self-similarity in multiparticle production. If the particle process exhibits self-similar behavior, then a modified form of G_q moment in terms of step function shows the remarkable power law dependence on phase space bin size.

Here an attempt has been made to study the multifractality for the emission spectra of projectile fragmentation data obtained from NIKFI-BR2 (JINR) emulsion stacks exposed to 4.5 AGeV ^{24}Mg beam.

Mathematical formalism

The particle number density in each rapidity bin depends on whether the resolution of the binning is of the order of or better than the rapidity separation between the neighboring particles. It has been found that if the resolution is of the order of the average separation of two neighboring particles in the phase space, then the binning of the phase space with that resolution may result in some empty bins. Considering the empty bins in the distribution are analogous to the holes then the set of non-empty bins would form a fractal set.

By introducing a step function to suppress the low multiplicity events for which the statistical fluctuation is large, Hwa and Pan [2] proposed a modified G_q moment to investigate the fractal properties of the emission spectra of different charged secondaries. In this investigation, the generalized moments G_q is first estimated in χ ($\cos\theta$) space using the relation,

$$G_q = \sum_{m=1}^{M} \left(\frac{n_m}{N_{eV}}\right)^q \theta\,(n_m - q) \qquad (1)$$

Here $\theta\,(n_m - q)$ is a step function such that $\theta\,(n_m - q) = 1$ for $n_m > q$, and $\theta\,(n_m - q) = 0$ for $n_m < q$.
For an ensemble of events, the averaging is done as

$$\langle G_q \rangle = \frac{1}{N_{eV}} \sum G_q \qquad (2)$$

A power law dependence of $\langle G_q \rangle$ on the phase space bin size, or on the number of phase space bins M, of the form represented by the equation $\langle G_q \rangle \propto M^{-\tau_q}$ indicates self similarity in the emission pattern. The exponent τ_q, called the fractal index, can be obtained from the asymptotic behavior.

To calculate the statistical contribution to $\langle G_q \rangle$, n particles are distributed randomly in the specified phase space. For each event, fractal moment is calculated with redistributed particles and $\langle G_q^{st} \rangle$ is obtained by using the equation 2.
The dynamical component of $\langle G_q \rangle$ is then estimated by using the following formula given by Chiu [3],

$$\langle G_q^{dyn} \rangle = \left(\frac{\langle G_q \rangle}{\langle G_q^{st} \rangle} \right) M^{1-q}$$

If $\langle G_q \rangle$ is purely statistical then,

$$\langle G_q^{dyn} \rangle = M^{1-q}$$

Under such condition, $\tau_q^{dyn} = q - 1$

Thus, any deviation of τ_q^{dyn} from $q - 1$ indicates that $\langle G_q \rangle$ contains dynamical information.

The self-similarity of a fractal object is characterized by the generalized dimension D_q which is now defined by the relation,

$$D_q = \frac{\tau_q^{dyn}}{q-1}$$

Results and discussion

Fractal analysis in the spatial distribution of projectile fragments:

To calculate the statistical contribution to $\langle G_q \rangle$, equal number of events are generated in $\cos\theta$ space by a random number generator with $\cos\theta$ values lying between -1 and +1. The $\cos\theta$ values are then converted into $\chi(\eta)$ values and $\langle G_q^{st} \rangle$ is then estimated using equation 2. The variation of $\ln \langle G_q \rangle$ with $\ln M$ for $q = 2$, 3 and 4 for experimental as well as for random number generated events are shown in Fig. 1. The solid lines are for the experimental data points and the dotted lines are for the equal number of generated events.

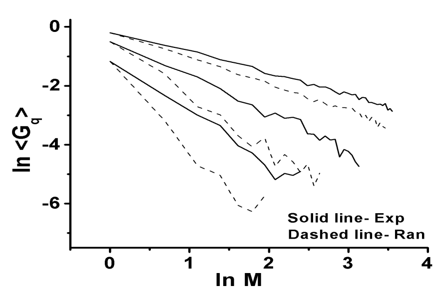

Fig.1 Variation of $\ln \langle G_q \rangle$ with ln M

Deviation of τ_q^{dyn} from $q - 1$ is plotted in Fig. 2. It indicates that G_q contains information about dynamical contribution to the fluctuations and this deviation is more as we go to the higher order of moments.

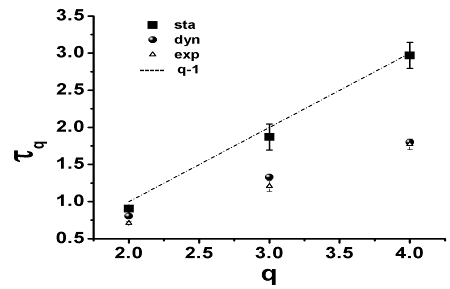

Fig. 2 Deviation from q-1

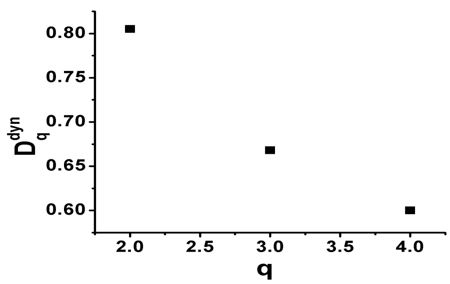

Fig.3 Variation of D_q with q

Variation of D_q with q is plotted in Fig. 3. It is clear from the figure that the generalized dimension D_q decreases linearly with q, indicating multifractality in the emission spectra of projectile fragments.

Acknowledgement: Financial support of UGC, India, in the form of a research fellowship is thankfully acknowledged.

References

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