Wavelet analysis in high multiplicity events at CERN SPS energy

Prabir Kumar Haldar*, Sanjib Kumar Manna
Dinhata College, Department of physics, Dinhata, Coochbehar, West Bengal, Pin -736135, India
Corresponding Author: prabirkrhaldar@yahoo.com*

Introduction

Wavelet analysis is the study of any function by expanding it in a wavelet series (or integrals). Due to the completeness of the system, it also allows for the inverse transformation (synthesis) to be done. This means that the original function or some parts of it containing the investigated correlations may be restored without any loss of information. In the analysis of non-stationary signals or inhomogeneous images (like modern paintings with very sharp figure edges), the locality property of wavelets leads to a substantial advantage over the Fourier transform, which provides only the knowledge of global frequencies (scales) of the object under investigation, because the system of functions used (sine, cosine or complex exponents) is defined over an infinite interval. Wavelet analysis reveals the local properties of any pattern in an individual event on various scales and, moreover, removes smooth polynomial trends and emphasizes fluctuation patterns. By choosing the strongest fluctuations, one may chuck out statistical fluctuations and observe those dynamic ones which exceed the statistical component.

In this paper we have chosen five events, having multiplicity 229, 228, 202, 190 &171 from ultra-relativistic nuclear interactions of ^{32}S- Ag/Br interactions at 200 A GeV/c and identified as unusual superspiky events, showing two distinct picks in the pseudo rapidity distribution and getting the nuclear refractive index almost close to unity **[1]**, using the concept of Cherenkov gluon radiation from the knowledge of ring-like events **[2]**. Here we have introduced the wavelet analyses technique in η space for the above said events. P Carruthers had the first attempt to use wavelet analysis in multiparticle production **[3-5]**. Commonly used wavelets form a complete orthonormal system of functions with a finite support and can be obtained from one another by dilations and translations. So, by changing the scale (dilations), they can distinguish the local characteristics of a signal on various scales and, by translations, they cover the whole region in which it is studied. The orthogonality of wavelet insures that the information on a definite resolution level (scale) does not interfere with other scale information. The wavelet method employed to study the angular spectra of produced particles provides useful mathematical tools to determine simultaneously positions and widths of irregularities which can be interpreted as particle collective flows. The another great advantage is that unlike Fourier analysis where only two basis functions exist, the wavelet analysis have an infinite set of possible basis functions. The wavelet basis should be chosen according to the properties of expected signals in order to ensure more direct and transparent access to the information. In general, the continuous wavelet transform of function *f(x)* has the form $W_\psi(a,b)f = \frac{1}{\sqrt{C_\psi}}\int_{-\infty}^{\infty} f(x)\psi_{a,b}(x)dx$, where x is a studied quantity and C_Ψ is a normalizing constant. The functions $\psi_{a,b}(x) = a^{-\frac{1}{2}}\psi(\frac{x-b}{a})$ are shifted and/or dilated derivations of mother wavelet function Ψ(x) characterized by translation parameter b and dilation parameter or scale a. The coefficients W_Ψ (a,b) can be interpreted as contributions (amplitudes) of wavelets $\Psi_{a,b}$ to spectrum f(x).

The choice of the wavelet depends on the problem studied and is not unique. As an example of continuous wavelets, let us mention the so-called `Mexican hat' wavelet, which is the second derivative of the Gaussian function ($g_2 = (1-x^2)e^{-\frac{x^2}{2}}$). In our analyses we use g_2 as the mother wavelet since the signals of approximately Gaussian-like shape are expected. We have chosen the variable x as η and the distribution of pseudorapidity is given by $f(\eta) = \frac{dn}{d\eta} = \frac{1}{N}\sum_{i=1}^{N}\delta(\eta-\eta_i)$, where N is the number of particles in a studied data sample and η_i is pseudorapidity of i^{th} particle. In **fig.1** we have shown the variation of the first derivative [**fig.1 (a)**] and the second derivative [**fig.1 (b)**] of Gaussian function with x. The application of the wavelet transform of pseudorapidy distribution

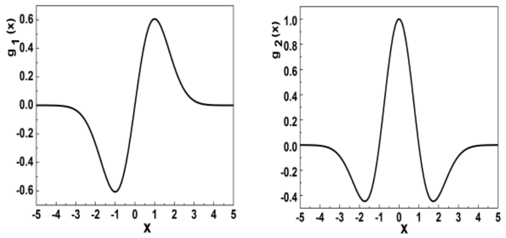

Fig.1 (a) **Fig.1 (b)**

of produced pions in high energy collision, allows one to investigate its behavior at various scales. This approach basically means that only the global features of the studied events are extracted but this is quite sufficient at the very first step. Wavelet g_2 pseudorapidity spectra for the event of multiplicity 229, said above, at two different scales *a* (0.2 & 0.5) are presented in **fig.2.** The maximums in the spectra in **fig.2** are related to preferred pseudorapidities of groups of secondary particles. The size of groups is indicated by the scales *a*. At the finest scales only the small particle groups are observed while at the coarse scales the large particle collective flows are clearly perceptible. The number of particles included in separate groups follows from the size of areas corresponding to the local maximums. The size of each area is indicated by its width and height of the maximum. The particles can be roughly assorted to the three main groups: 1. the target fragmentation region at low pseudorapidities, 2. the projectile fragmentation region at high pseudorapidities, 3. the central region at medium pseudorapidities.

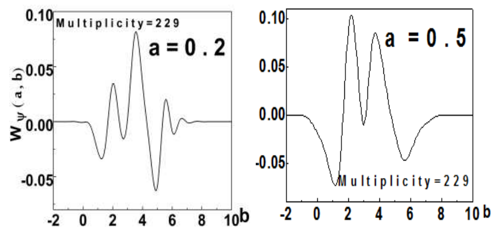

Fig.2

We have studied the Wavelet g_2 pseudorapidity spectra for rest of the four events and same nature of the graph is observed except the size of areas of the local maximums. When combining many plots like those in **fig.2**, three-dimensional graph can be constructed illustrating the dependence of W_Ψ (a, b) coefficients on pseudorapidity *b* and scale *a*. It is shown for the event of multiplicity 229 for experimental data, in **fig.3**. The local maximums in the wavelet spectrum interpreted as the preferred pseudorapidities of particle groups seen at the characteristic scales. Moving along the scale axis, the evolution of clusterization of particles can be examined.

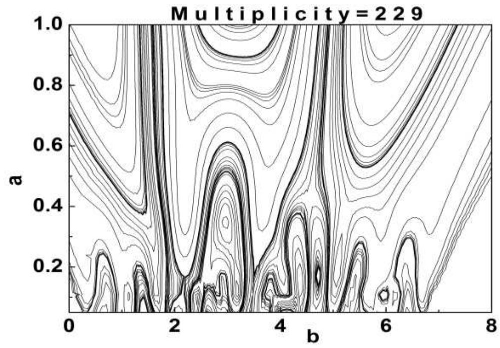

Fig.3

Fig.3 also indicates the most dominant scales since the maximums corresponding to groups of particles are visible mainly in the scale range from **0.01 to 0.6.** For the other events it is observed that the nature of the graph is different for different event with different characteristic scale.

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