On the soft gluon radiation in partonic plasma

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The process $\mathrm{g}+\mathrm{g}\to\mathrm{g}+\mathrm{g}+\mathrm{g}$ plays an important role in the phenomenology of heavy ion collisions at relativistic energies. The spectrum of the emitted gluon for the above process was considered by Gunion and Bertsch in [1]. Recently attempts have been made to revisit/generalize the GB formula [2, 3]. The number non-conserving process, $\mathrm{g}+\mathrm{g}\to\mathrm{g}+\mathrm{g}+\mathrm{g}$ has drawn particular attention because of its role in (i) chemical equilibration in the deconfined phase of quarks and gluons, (ii) energy loss of fast gluons propagating through Quark Gluon Plasma (QGP), (iii) evaluation of transport co-efficients etc. In the present work we evaluate the spectra of emitted gluons from the reaction $\mathrm{g}+\mathrm{g}\to\mathrm{g}+\mathrm{g}+\mathrm{g}$ by relaxing some of the approximations adopted earlier and investigate the effects of corrections to the widely used GB formula [4]. Particularly, we will inspect the effects of the correction terms on physical quantities like the equilibration time and the energy loss of fast gluon in QGP.

The square of the invariant amplitude for the process $\mathrm{g}(k_1)+\mathrm{g}(k_2)\to\mathrm{g}(k_3)+\mathrm{g}(k_4)+\mathrm{g}(k_5)$, $|M_{\mathrm{gg}\to\mathrm{ggg}}|^2$ can be written as [4]:

$$\begin{aligned}|M|^2_{\mathrm{gg}\to\mathrm{ggg}} &= 12g^2|M_{\mathrm{gg}\to\mathrm{gg}}|^2\frac{1}{k_\perp^2}\\ &\times\ [(1+\frac{t}{2s}+\frac{5t^2}{2s^2}-\frac{t^3}{s^3})\\ &-\ (\frac{3}{2\sqrt{s}}+\frac{4t}{s\sqrt{s}}-\frac{3t^2}{2s^2\sqrt{s}})k_\perp\\ &+\ (\frac{5}{2s}+\frac{t}{2s^2}+\frac{5t^2}{s^3})k_\perp^2], \qquad (1)\end{aligned}$$

where $k_\perp$ is the transverse momentum of the radiated gluon, s, t are Mandelstam variables for the process $\mathrm{gg}\to\mathrm{gg}$. The terms $\mathcal{O}(k_\perp^{-1})$ and $\mathcal{O}(k_\perp^0)$ appearing in Eq. 1 will be henceforth called the correction terms. We will demonstrate that the contributions from these terms are non-negligible and will have crucial importance for the phenomenology of heavy ion collisions at ultra-relativistic energies.

The spectra of the emitted gluon can be written as:

$$\begin{aligned}\frac{dn_g}{d^2k_\perp d\eta} &= \left[\frac{dn_g}{d^2k_\perp d\eta}\right]_{GB}[(1+\frac{t}{2s}+\frac{5t^2}{2s^2}-\frac{t^3}{s^3})\\ &-\ (\frac{3}{2\sqrt{s}}+\frac{4t}{s\sqrt{s}}-\frac{3t^2}{2s^2\sqrt{s}})k_\perp\\ &+\ (\frac{5}{2s}+\frac{t}{2s^2}+\frac{5t^2}{s^3})k_\perp^2)], \qquad (2)\end{aligned}$$

The equilibration rate of gluons for the process $\mathrm{gg}\longrightarrow\mathrm{ggg}$ can be obtained once we know the differential cross section of this process. The radiative cross section can be obtained by integrating the following expression:

$$\frac{d\sigma^{2\longrightarrow 3}}{d^2k_\perp d\eta d^2q_\perp}=\frac{d\sigma^{2\longrightarrow 2}}{d^2q_\perp}\frac{dn_g}{d^2k_\perp d\eta}, \qquad (3)$$

where $q_\perp$ is the momentum transfer and η is the rapidity of the emitted soft gluon. The ratio of the equilibration rate obtained by using present gluon distribution to that using GB spectrum is displayed in Fig.1. We evaluate the equilibration rate for this process with $s = 18T^2$ with and without the correction terms. The equilibration rates obtained from the spectra of Refs. [2], [3] and present work normalized by the GB spectra (Γ_R) are displayed in Fig. 1. We observe that the equilibration rate obtained with the correction terms is smaller at high temperature ($\sim$ 500 MeV) compared to the scenario when the corrections are neglected. This indicates that the contribution from the correction terms help in enhancing the equilibration in the gluonic system.

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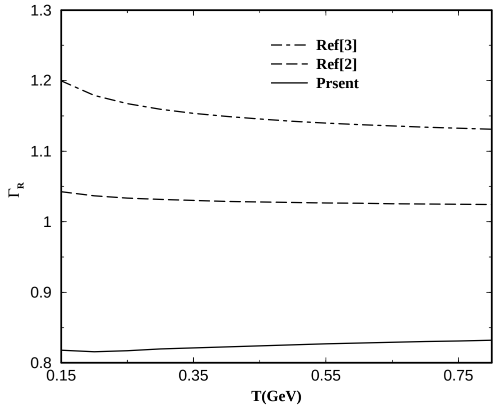

FIG. 1: Temperature variation of the ratio of the equilibration rate (inverse of the time scale) obtained in the present work (solid line), Refs. [2] (dashed line) and [3] (dot-dashed) normalized by the GB value for the process gg → ggg.

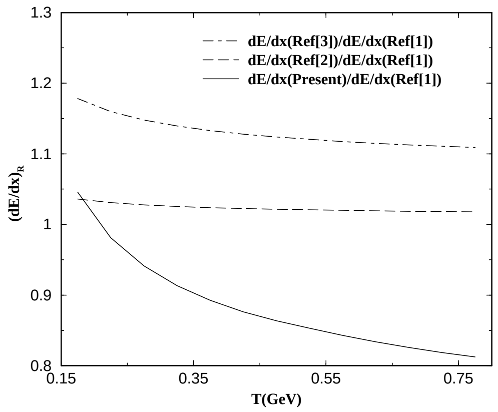

FIG. 2: Energy loss of fast gluon in a gluonic fluid. E = 5 GeV

The energy loss of a gluon passing through the QGP medium can now be calculated using the gluon spectrum of Eq. 2. The energy loss per collision can be estimated as:

$$\epsilon = \int d^2k_\perp d\eta \frac{dn_g}{d^2k_\perp d\eta} k_0 \theta(\Lambda^{-1} - \tau_F) \times \theta(E - k_\perp cosh\eta), \quad (4)$$

where $k_0 = k_\perp cosh\eta$ is the energy of the radiated gluon and τ_F is the formation time of the gluon. The first θ-function in Eq. 4, involving Λ^{-1} or interaction time, is introduced for the Landau-Pomeranchuk-Migdal (LPM) effect. The LPM effect imposes restriction on the phase space of the radiated gluon, it must have $\tau_F(= cosh\eta/k_\perp)$ less than the mean free time, Λ^{-1}. The gluon can be emitted for time scale larger than τ_F. The second θ-function sets the upper limit for the energy of the radiated gluon.

To proceed further, we replace $q_\perp^2$ by its average value evaluated using the formula:

$$\langle q_\perp^2 \rangle = \frac{1}{\sigma_{el}} \int_{m_D^2}^{\frac{s}{4}} dq_\perp^2 \frac{d\sigma_{el}}{dq_\perp^2} q_\perp^2, \quad (5)$$

where σ_{el} is the cross section for the elastic process: gg → gg.

In contrast to previous works [2, 3] the value of s is taken as $s \sim 6ET$ allowing the possibility for the incident gluon to remain out of thermal equilibrium. With all the above ingredients we are now ready to evaluate energy loss (dE/dx) of a fast gluon in the QGP as follows:

$$-\frac{dE}{dx} = \epsilon \cdot \Lambda. \quad (6)$$

The energy loss obtained from Eqs. 2, 4 and 6 normalized by that resulting from GB approximation is displayed in Fig. 2. We find that the magnitude of the energy loss with the current gluon spectrum (Eq. 2) is smaller than the one obtained with spectrum of Refs. [2] and [3]. With the correction terms the value is down by about 30% at high temperature. Such differences may have crucial consequences on the heavy ion phenomenology at RHIC and LHC collision energies.

References

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