Shear viscosity of a pion gas

Sukanya Mitra,* Santosh K Das, Sourav Sarkar, and Jan-e Alam
Theoretical Physics Division, Variable Energy Cyclotron Centre, 1/AF, Bidhan Nagar , Kolkata - 700064

The experimental data from RHIC, especially the measured elliptic flow indicate that the matter produced in Au+Au collisions exhibit properties which are more like a strongly interacting liquid than a weakly interacting gas. The shear viscosity η or the internal friction of the fluid symbolizes the ability to transfer momentum over a distance of $\sim$ mean free path. Therefore, in a system where the constituents interact strongly the transfer of momentum is performed easily - resulting in lower values of η. Consequently such a system may be characterized by a small value of η/s. The importance of viscosity also lies in the fact that it damps out the variation in the velocity and make the fluid flow laminar. A very small viscosity (large Reynold number) may make the flow turbulent.

Although a large amount of work has been done on shear viscosity in QGP phase, the shear viscosity in hadronic matter has received much less attention so far. In the present work shear viscosity has been evaluated in a kinetic theory approach by solving Boltzmann transport equation using the relaxation time approximation.

When a system is slightly away from equilibrium collisions within the system attempts to restore it back. These collisions involve momentum transfer between different elements of the system which sets up different dissipative processes. These viscous effects modify the energy-momentum stress tensor of the fluid. To first order in velocity gradients this is given by,

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} - \eta\langle\nabla^\mu u^\nu\rangle \quad (1)$$

where only the effect of shear viscosity has been considered. The presence of a velocity gradient between adjacent layers of fluid results in a distortion in the distribution of components making up the fluid element; modifying it by an amount δf. Incorporating this viscous modification of the distribution function in the integral form of $T^{\mu\nu}$ one gets,

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\nu \ (f_0 + \delta f) \quad (2)$$

where f_0 is the equilibrium distribution function. Comparing the above two equations, the shear viscosity can be expressed in terms of δf as

$$\eta\langle\nabla^\mu u^\nu\rangle = -\int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\nu \ \delta f \quad (3)$$

Again, an expression for δf can be obtained from the Boltzmann transport equation by expressing the collision term in terms of the relaxation time τ,

$$\delta f = -\tau \frac{f_0(1+f_0)}{TE} p_i p_j (\nabla^i u^j)_{trl} \quad (4)$$

where,

$$(\nabla^i u^j)_{trl} = \frac{1}{2}(\frac{\partial V_j}{\partial x_i} + \frac{\partial V_i}{\partial x_j} - \frac{2}{3}\partial_k V^k \delta_{ij}) \quad (5)$$

The coefficient of shear viscosity thus obtained, can be written for a multicomponent system as,

$$\eta = \frac{1}{15T}\sum_a \int d\Gamma_a \frac{p_a^4}{E_a^2} f_0^a (1 + f_0^a)\tau_a \quad (6)$$

where $d\Gamma = g\frac{d^3p}{(2\pi)^3}$ and g denotes the number of degrees of freedom. The summation is over the various components of the system. The relaxation time for the particle i is given by

$$\tau_i^{-1} = \sum_j \ \tau_{ij}^{-1} \quad (7)$$

*Electronic address: sukanyam@vecc.gov.in

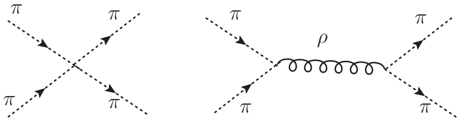

FIG. 1: Feynman diagrams for $\pi\pi$ scattering

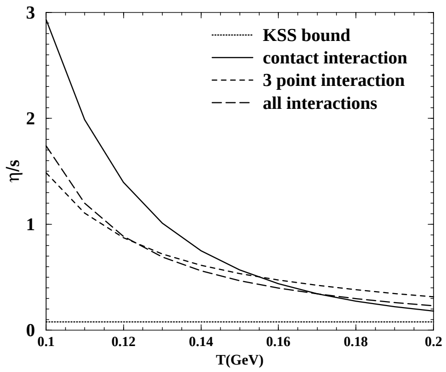

FIG. 2: The variation of η/s with temperature for pion gas using lowest order chiral interactions.

For a pion gas that we are considering, i, j denote the charge states π^+, π^- and π^0. The scattering time τ_{ij} is given by the inverse of the rate of elastic scattering of the i^{th} particle with the j^{th} species in the medium. For a reaction $ij \to kl$ this is defined as

$$\Gamma_i = \frac{g_i}{2E_i} \prod_{\alpha=j,k,l} \frac{g_\alpha d^3p_\alpha}{(2\pi)^3 2E_\alpha} f_0^j(1+f_0^k)(1+f_0^l) \times \overline{|M|^2}\,(2\pi)^4\delta^4(p_i+p_j-p_k-p_l)\ . \quad (8)$$

For the evaluation of the invariant amplitudes for scattering between pions we have considered both contact interactions as well as processes where a ρ meson is exchanged (Fig. 2). The interaction Lagrangian used for this purpose is obtained from chiral perturbation theory and is given by

$$\mathcal{L}_{\pi\pi} = -\tfrac{1}{6f_\pi^2}[(\partial_\mu\vec{\pi}.\partial^\mu\vec{\pi})(\vec{\pi}.\vec{\pi}) - (\vec{\pi}.\partial_\mu\vec{\pi})(\vec{\pi}.\partial^\mu\vec{\pi})] + \tfrac{m_\pi^2}{24f_\pi^2}(\vec{\pi}.\vec{\pi})^2 + g_\rho\vec{\rho}_\mu.(\vec{\pi}\times\partial^\mu\vec{\pi}) \quad (9)$$

where $f_\pi = 93$ MeV is the pion decay constant and s, t, u are the Mandelstam variables.

For the contact diagrams the scattering amplitude is given by,

$$\overline{|M|^2} = 2\{-\frac{s}{f_\pi^2}+\frac{2m_\pi^2}{f_\pi^2}\}^2 + 2\{-\frac{u}{f_\pi^2}+\frac{2m_\pi^2}{f_\pi^2}\}^2 + 4\{-\frac{t}{f_\pi^2}+\frac{m_\pi^2}{f_\pi^2}\}^2 + \{\frac{m_\pi^2}{f_\pi^2}\}^2 \quad (10)$$

and the corresponding amplitude for the ρ exchange diagrams is,

$$\overline{|M|^2} = 6g^4\{\tfrac{(u-t)^2}{(s-m_\rho^2)^2+\Gamma^2m_\rho^2}\} + 4g^4\{\tfrac{(s-t)^2}{(u-m_\rho^2)^2+\Gamma^2m_\rho^2}\} + 2g^4\{\tfrac{(s-u)^2}{(t-m_\rho^2)^2+\Gamma^2m_\rho^2}\} + 4\tfrac{(s-u)(t-u)\{(s-m_\rho^2)(t-m_\rho^2)+\Gamma^2m_\rho^2\}}{\{(s-m_\rho^2)^2+\Gamma^2m_\rho^2\}\{(t-m_\rho^2)^2+\Gamma^2m_\rho^2\}} + 8\tfrac{(u-t)(s-t)\{(s-m_\rho^2)(u-m_\rho^2)+\Gamma^2m_\rho^2\}}{\{(s-m_\rho^2)^2+\Gamma^2m_\rho^2\}\{(u-m_\rho^2)^2+\Gamma^2m_\rho^2\}} \quad (11)$$

where Γ is the rho decay width which has the form,

$$\Gamma = \frac{g_\rho^2 m_\rho}{48\pi}(1-4\frac{m_\pi^2}{m_\rho^2})^{3/2}\ . \quad (12)$$

The entropy density of the gas is then evaluated using the well-known formula

$$s = -\sum_a \int \frac{d^3p_a}{(2\pi)^3} f_0^a \ln f_0^a \quad (13)$$

Fig. 2 shows the variation of the ratio η/s with temperature for the contact interaction, three point interaction as well as for the total. In all the three case, η/s decreases with respect to temperature and is well above the quantum bound $1/4\pi$.