Systematics of triaxial moment of inertia and deformation parameters (β, γ) in even-even nuclei of mass region A = 90-120

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Introduction:

The deformation parameter β and γ of the collective model of Bohr and Mottelson [1, 2] are basic descriptors of the nuclear equilibrium shape and structure. In recent past the sets of deformation parameters (β, γ) have been extracted from both level energies and E2 transition rates in even Xe, Ba and Ce nuclei (A~120-140) and Hf, W, Os, Pt and Hg nuclei (A~160-200) using rigid triaxial rotor model of Davydov – Filippov (DF) [2-4]. Researcher have found that the values of β obtained separately from energy and transition rate (β_e & β_b respectively), though, are found almost equal in heavy mass region (A ~160-200) but, not so in medium mass (A~120-140) nuclei. This observation puts a question mark whether the β dependence of moment of inertia in hydrodynamic model is reliable [4].

The purpose of the present work is to study a relatively lighter mass region (A~90-120) where the gap between values of two sets of β may further increase. To improve the calculations for extracting β_e, the use of Grodzins rule will be made along with uncertainties [5], since only through this rule the $E2_1^+$ is related with β_G (value of β for symmetric nucleus and evaluated using Grodzins rule).

Procedure for evaluating deformation parameters (γ, β) and moment of inertia:

The asymmetric parameter (γ) may easily be evaluated by experimental energy ratio of $E2_2^+$ and $E2_1^+$ states using DF model [2]. The reduced transition probabilities in DF model given by

$$B\left(E2; 2_{1,2}^+ \rightarrow 0_1^+\right) = \frac{e^2Q_0^2}{32\pi}\left[1 + (-1)^{\sigma_{1,2}}\frac{3-2X}{\sqrt{9-8X}}\right] \quad (1)$$

Where, Q_0 is defined as in the axially symmetric case by

$$Q_0 = \frac{3ZR^2\beta}{\sqrt{5\pi}} \quad (2)$$

The quadrupole deformation parameter (β) is easily determined from equation (1) and (2) for the $(2_1^+ \rightarrow 0_1^+)$ transition which is often known in experiment and we denote the parameter by β_b. In a second way β is estimated by using the Grodzins empirical rule [3] for a symmetric nucleus using relation

$$B(E2; 2_1^+ \rightarrow 0_1^+).E2_1^+ = (2.5 \pm 1) \times 10^{-3}Z^2 \quad (3)$$

Equation (1), (2) and (3) give the value of β for symmetric nucleus ($=\beta_G$). These values are calculted with uncertainties. The value of β needs to be corrected by the factor $\hbar^2/J_0$ for asymmetric nucleus. There are three values of β corresponding to the values of β_G

$$\beta = \beta_G\left(\frac{X}{9-\sqrt{81-72X}}\right)^{\frac{1}{2}} \quad (4)$$

We denote these parametrs by β_{e^-}, β_e, β_{e^+}(e.g. β_e with ± 1 uncertainty) corresponding to numeriacls 1.5, 2.5 and 3.5 respectively used in Grodzins rule.

The moment of inertia is extracted from equation (5) using the enrgy of first excited 2[+] state and is denoted by I_0 given by equation (5)

$$E_{2\,1,2}^+ = \frac{9\left[1-(-1)^{\sigma_{1,2}}\sqrt{1-\frac{8}{9}X}\right.}{X} \quad (5)$$

In the units of $\hbar^2/J_0$ where, $\sigma_{1,2}$ = 0, 1 and $X = \sin^2 3\gamma$

$$I_0 = \frac{J_0}{\hbar^2} = \frac{9-\sqrt{81-72X}}{XE_1^+} \quad (5)$$

Discussion:

The deformation parameters β_e with ± 1 uncertainty and β_b extracted from energies (β_{e^-}, β_{e^+} and β_b) and E2 transition rates for Mo, Ru and Pd nuclei are plotted with neutron numbers in fig 1. It is found that the deformation parameter β increases with neutron number in general and this trend is expected since the deformation gets saturated at midshell (N = 66) [6] and the value of β begins decreasing with increase of N beyond midshell as depicted in Pd nuclei. The value of β_b lies between β_{e^+} and β_{e^-}in all of the nuclei under consideration. Also β_b and β_{e^-} are much closer in Mo & Pd nuclei.

Another plot of moment of inertia (I_0) versus deformation (β_b) is drawn (fig. 2) where β and I_0 are found related significantly. The moment of inertia I_o increses with N_pN_n in general, just like β_b. This, justifies the β – dependence of moment of inertia in asymmetric nucleus.

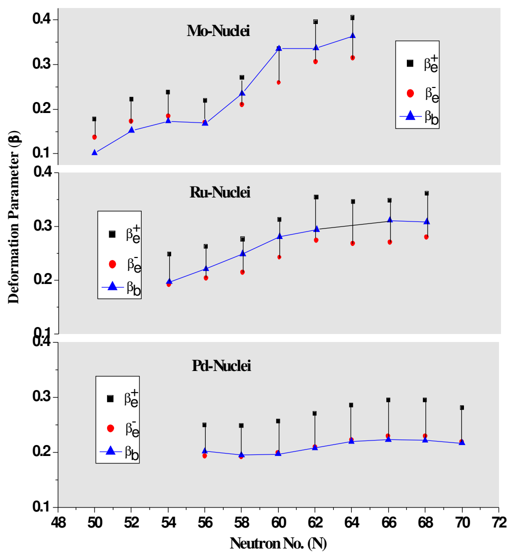

Fig. 1

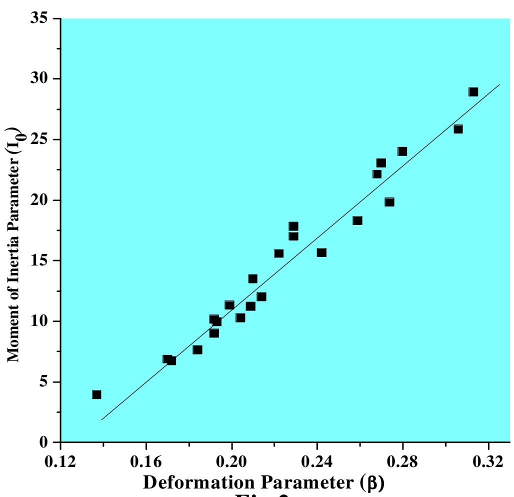

Fig.2

The Grodzins empirical rule fits for well known E2 and B(E2) values are used to predict $B(E2; 0_1^+ \rightarrow 2_1^+)$ and $B(E2; 4_1^+ \rightarrow 2_1^+)$ for $^{84\text{-}90}$Mo, $^{88\text{-}94,106,114}$Ru and $^{94\text{-}100,118}$Pd nuclei, where transition energies are not known. The predicted values of B(E2)s are tabulated in Table III along with earlier predictions of Bohr-Mottelson and Grodzins [17] for the sake of comperison. One can feel confident in the model by getting convenced with internal consistency in triaxial relations and one can go ahead in search of a possibility of simple one parameter relationship in γ, β with N_pN_n in a wider spectrum of nuclear chart

Conclusions:

(a) β values extracted from B(E2) are closer to the values evaluated from $E2_1^+$ using numerical coefficient 1.5×10^{-3} in Grodzins rule however, all the values of β_b lie in between the values of β obtained from $E2_1^+$ using numerical values 1.5×10^{-3} and 2.5×10^{-3} in empirical relation of Grodzin's (β_e- $> \beta_b > \beta_e$).

(b) There is a good correlation in β and $\frac{J_0}{\hbar^2}$ in the mass region A~90 – 120. In a broader perspective, the present work not only suggests a procedure for obtaining closer value of β evaluated seperately from energy and transition rate but also extends a support to smooth dependence of β on moment of inertia in hydrodynamic model against the general beleif [4].

Table – I

Nucl.	$B(E2; 0_1^+ \rightarrow 2_1^+)$			$B(E2; 4_1^+ \rightarrow 2_1^+)$
	*Bohr Mottels	*Grodzins	Present work	Present work
^{84}Mo	-	-	0.297	0.839
^{86}Mo	-	-	0.253	0.0723
^{88}Mo	0.29(12)	0.32(11)	0.213	0.0603
^{90}Mo	0.28(12)	0.31(10)	0.194	0.0547
^{88}Ru	-	-	0.275	0.0778
^{90}Ru	-	-	0.248	0.069
^{92}Ru	0.34(14)	0.36(12)	0.232	0.0646
^{94}Ru	0.20(8)	0.21(7)	0.181	0.0503
^{114}Ru	-	-	2.456	0.0711
^{94}Pd	-	-	0.258	0.0775
^{96}Pd	0.22(9)	0.23(8)	0.201	0.0555
^{98}Pd	0.36(15)	0.37(12)	0.263	0.0811
^{100}Pd	0.47(19)	0.47(16)	0.358	0.1001
^{118}Pd	-	-	0.729	0.2063

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