A Simple approach of extracting deformation parameter β for asymmetric nuclei from $E2_1^+$

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Introduction

It has been observed that the values of β calculated from energy and transition rate were not closer [1, 2]. The reason for this gap in the values of β was assigned to not fully correct β – moment of inertia relation of hydrodynamic model. In the present work, a simple approach is proposed to evaluate β for asymmetric nucleus from $E2_1^+$ using the semi-empirical relation of Grodzins [3].

I. Procedure for evaluating β from $E2_1^+$:

(a) Review of Previous Work:

Semi empirical relation of Grodzins for a nucleus that relates $E2_1^+$ with $B(E2; 2_1^+ \to 0_1^+)$, is given

$$E2_1^+.B(E2; 2_1^+ \to 0_1^+) = (2.5 \pm 1) \times 10^{-3} Z^2 A^{-1} \quad (\text{MeV.e}^2\text{b}^2) \tag{1}$$

For symmetric nucleus it has been modified as –

$$E2_1^+.B(E2; 2_1^+ \to 0_1^+) = 2.5 \times 10^{-3} Z^2 A^{-1} \tag{2}$$

Using equation (2), the moment of inertia parameter J_0 is related to an effective value of β [4] for symmetric nucleus

$$\frac{6\hbar^2}{2J_0} = \frac{1224}{\beta^2 A^{7/3}} \tag{3}$$

The hydrodynamic relation [4] relates $E2_1^+$ with moment of inertia J_0 and asymmetric parameter γ for asymmetric nucleus (γ ≠ 0) as

$$E2_1^+ = \frac{6\hbar^2}{2J_0} \frac{9-\sqrt{81-72\sin^2 3\gamma}}{4\sin^2 3\gamma} \tag{4}$$

For symmetric nucleus (γ = 0), equation (4) reduces to

$$E2_1^+ = \frac{6\hbar^2}{2J_0} \tag{5}$$

Equation (3) and (5) gives a relation between $E2_1^+$ and β for symmetric nucleus as –

$$E2_1^+ = \frac{1224}{\beta^2 A^{7/3}} \tag{6}$$

From equation (3) and (4), we can get

$$E2_1^+ = \frac{1224}{\beta^2 A^{7/3}} \frac{9-\sqrt{81-72\sin^2 3\gamma}}{4\sin^2 3\gamma} \tag{7}$$

Equation (7) allows us to extract β from $E2_1^+$ for asymmetric nucleus.

(b) Present Approach:

We have evaluated the value of Grodzins product $A.E2_1^+.B(E2; 2_1^+ \to 0_1^+)/Z^2 \ (= C)$ for the nuclei under consideration. The value varies from 1.5 to 4.5 (× 10^{-3}) in the mass region taken in the present work. For most of the nuclei, the value of C is more than 2.5 except $^{200-202}$Hg nuclei. As such the nuclei taken in the present work are asymmetric possessing $\beta > 0$ and $0 < \gamma < 60^0$. Therefore, equation (6) should become valid for asymmetric nucleus if 2.5 of the Grodzins product is replaced by the exact value C. Thus, equation (6) will be rewritten for asymmetric nucleus as –

$$E2_1^+ = \frac{1224 \times (C/2.5)}{\beta^2 A^{7/3}} \tag{8}$$

On comparison equation (7) and (8), we find that

$$\frac{C}{2.5} = \frac{9-\sqrt{81-72\sin^2 3\gamma}}{4\sin^2 3\gamma}$$

This is justified for symmetric nucleus. Also since for C=2.5, the value of asymmetric parameter γ is zero. We can assume safely that $C/2.5$ takes care of the factor $\frac{9-\sqrt{81-72}\sin^2 3\gamma}{4\sin^2 3\gamma}$. Thus, the controversy of the factor used in earlier work that produces unwanted values of β has been avoided.

Discussions:

The value of Grodzins constant C has been evaluated for W, Os, Pt and Hg nuclei using semi empirical relation -

$$C = A\, E2_1^+.B(E2; 2_1^+ \to 0_1^+)\, Z^{-2} \times 10^3$$

feeding known values of $E2_1^+$, atomic number Z, mass number A and $B(E2; 2_1^+ \to 0_1^+)$ and are listed in column 4 of Table I. β_e of the present work is evaluated using equation (8) and are listed in column 7 of Table I. β_b and β_e values have been taken from the work of ref. [1,2]. Variation in percentage of new and old β_b with β_e are written in column 8 of Table I.

Following observations are critical-

(a) Present values of β_e are closer to the values of β_b in whole of the mass spectrum.

(b) Maximum variation in the value of β_e from β_b is 56.6as noted by previous workers for ^{202}Hg nucleus, while in present work it goes to 12.1 only for ^{186}Os nucleus.

(c) Few anomalous observations in old work where $\beta_b \geq \beta_e$ and consequently the variation is either zero or negative are absent in present approach. The variation in $^{200-202}$Hg were too large in old values but are reasonably small in present values, although the values of Grodzins constant C, is much less than 2.5.

Conclusions: In this work, the quadrupole deformation β evaluated from $B(E2)$ and $E2_1^+$ are made closer adopting a new and simple approach by taking the exact value of Grodzins constant C in place of 2.5, which is taken for symmetric nucleus in the previous work. This is relevant to mention that the values of γ, obtained earlier by Gupta et al [5], extracted from $B(E2;2_1^+ \rightarrow 0_1^+)$, $E2_1^+$, Z and A using Grodzins relation has also shown the desired results for triaxial Samarium nuclei.

Table I.

Grodzins constant C, presently calculated β_e and variation in percent are listed in column 4, 7, 8 and 9. Data of column 2, 3, 5 and 6 are taken from reference [1, 2]

Nucl.	$E2_1^+$	$B(E2;2_1^+ \rightarrow 0_1^+)e^2b^2$	C	β_b	β_e [3,4]	β_e Present work	Percentage variation in β_e from β_b	
							Present	Old
^{178}W	106.1	-	2.8	-	0.27	0.28	-	-
^{180}W	103.6	0.838(46)	2.85	0.256(9)	0.27	0.28	9.3	4.5
^{182}W	100.1	0.830(2)	2.76	0.254(4)	0.27	0.28	10.2	6.3
^{184}W	111.2	0.746(14)	2.78	0.238(3)	0.25	0.256	7.5	5.0
^{186}W	122.6	0.688(12)	2.86	0.229(4)	0.24	0.245	7.0	4.8
^{174}Os	158.6	-	2.8	-	0.23	0.23	-	-
^{176}Os	135.0	-	2.8	-	0.25	0.25	-	-
^{178}Os	132.2	-	2.8	-	0.24	0.24	-	-
^{180}Os	132.4	-	2.8	-	0.24	0.24	-	-
^{182}Os	126.9	0.762(66)	3.04	0.238	0.24	0.254	6.7	0.8
^{184}Os	119.7	0.640(3)	2.44	0.216	0.24	0.23	6.5	11.1
^{186}Os	137.2	0.582(20)	2.57	0.205	0.23	0.23	12.1	12.2
^{188}Os	155.0	0.508(12)	2.56	0.192	0.22	0.21	9.4	14.5
^{190}Os	1.86.7	0.460(2)	2.82	0.182	0.21	0.20	9.8	15.4
^{192}Os	205.8	0.410(1)	2.80	0.170	0.20	0.19	11.7	17.6
^{194}Os	(218.5)	-	2.8	-	0.19	0.18	-	-
^{180}Pt	0.153	-	3.50	-	0.23	0.25	-	-
^{182}Pt	0.155	-	3.50	-	0.23	0.25	-	-
^{184}Pt	0.163	0.790(3)	3.89	0.237	0.22	0.25	5.5	-7.1
^{186}Pt	0.192	0.596(22)	3.49	0.200	0.21	0.22	10.0	5.0
^{188}Pt	0.266	0.520(9)	4.26	0.187	0.18	0.20	6.9	-3.7
^{190}Pt	0.296	0.350(4)	3.23	0.150	0.17	0.16	6.7	13.3
^{192}Pt	0.316	0.382(12)	3.81	0.155	-	0.167	7.7	-
^{194}Pt	0.328	0.332(12)	3.48	0.144	-	0.155	7.6	-
^{196}Pt	0.356	0.280(1)	3.21	0.131	-	0.141	7.6	-
^{198}Pt	0.407	0.212(10)	2.61	0.113	-	0.117	3.5	-
^{188}Hg	0.413	-	3.0	-	0.1<	0.14	-	-
^{190}Hg	0.416	-	3.0	-	0.14	0.13	-	-
^{192}Hg	0.423	-	3.0	-	0.14	0.13	-	-
^{194}Hg	0.428	-	3.0	-	0.14	0.13	-	-
^{196}Hg	0.426	0.230(1)	3.0	0.117	0.13	0.12	2.5	11.1
^{198}Hg	0.412	0.198(2)	2.52	0.108	0.13	0.11	1.8	20.3
^{200}Hg	0.367	0.171(2)	1.98	0.101	0.14	0.108	6.9	38.6
^{202}Hg	0.439	0.122(2)	1.69	0.083	0.13	0.088	6.0	56.6

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