N-N t - Matrix Effective Interaction for Finite Range (p, 2p) Reaction Calculations

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Similar to the (α, 2α) and (^{12}C, $2^{12}C$) cluster knockout reactions [1–3] a proper understanding of the (p, $2p$) reactions involves an understanding of the free p-p scattering in terms of the realistic p-p interaction[4]. Recent advances in the field of (p, $2p$) reactions incorporating finite range (FR)-DWIA formalism used p-p t-matrix effective interactions (t-MEI) which were ad hoc[5]. These effective interactions were based on the first order Born approximation and were essentially of the M3Y form of the type shown in Fig. (1). These contained energy dependent central, tensor as well as spin-orbit terms along with separate exchange contributions.

As we require a local t-MEI for our FR-DWIA calculations we attempt to generate these t-MEI from the basic definition[6] in its local form as,

$$t\Phi = V\Psi \quad (1)$$

where Ψ and Φ are solutions of the free N-N scattering Schrödinger equation (with proper boundary conditions) with and without the N-N interaction V, respectively. The strong short-range repulsion in the realistic N-N interaction makes an ad hoc perturbative solution of Eq.(1) unreliable.

In our approach the N-N t-MEI is obtained by solving Eq.(1) non-perturbatively using Reid soft core realistic interaction which fit the N-N scattering phase shift data, at various energies. The triplet states have coupled equations with tensor interaction, thus the coupled $^3S_1$ - $^3D_1$ and $^3P_2$ - $^3F_1$ scattering state wave functions are obtained by transformation method adopted for the deuteron ground state and p-p scattering [7–9]. In Fig.(2) the phase shifts as a function of E_{Lab} are seen to match with the experimental values upto ~ 300 MeV very nicely. Using these Ψ's the $t^{S,T}(E,\vec{r})$'s in the local form are,

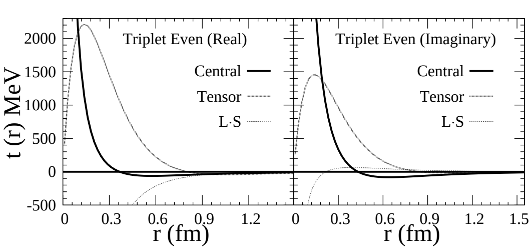

FIG. 1: Franey and Love t- matrix effective interaction for $t(r)$ S=1 and even ℓ at 200 MeV [5].

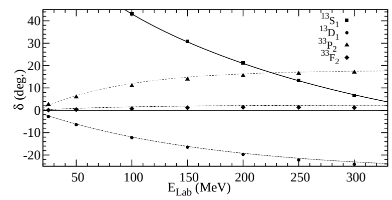

FIG. 2: Phase shifts, δ vs E_{Lab} for the $^{(2T+1)(2S+1)}\ell_J$ states, obtained for the Reid Soft core potential [4], compared with the experimental values.

$$t^{S,T}(E,\vec{r}) = \sum_{L=0,1,2,...} t_L^{S,T,E}(r) P_L(\hat{r})$$
$$= e^{\imath k_i z} V_\ell^{S,T}(\vec{r}) \Psi^{S,T}(\vec{k}_i,\vec{r})$$

where

$$t_L^{S,T,E}(r) = (L+\frac{1}{2}) \sum_{\ell,n} V_\ell(r) \imath^{\ell-n} (2\ell+1)(2n+1)$$
$$e^{\imath\sigma_\ell} \frac{u_{\ell(kr)}}{kr} \jmath_n(kr) \int_{-1}^{+1} P_L(t) P_\ell(t) P_n(t) dt.$$

Here $\jmath_n(kr)$ is a spherical Bessel function, $P_\jmath(t)$ is a Legendre polynomial, σ_ℓ is the Coulomb phase shift and $u_\ell(kr)$ is the radial scattering state wave function for ℓ^{th}-partial wave.

Fig.(3) shows $t_L^{S,T,E}(r)$'s for two representative energies of $E_{Lab} = 200$ MeV and 30 MeV and selected significantly large results out of the $^{13}S_0$, $^{11}P_1$, $^{13}D_0$, $^{33}P_0$, $^{31}S_1$, $^{33}P_1$, $^{13}P_2$, $^{31}D_1$, $^{31}D_2$ and $^{33}F_2$ channel state. One makes some general observations.

(1) For every (S,T,ℓ) there is a large number of L's contributing to the $t^{S,T,E}(\vec{r})$.

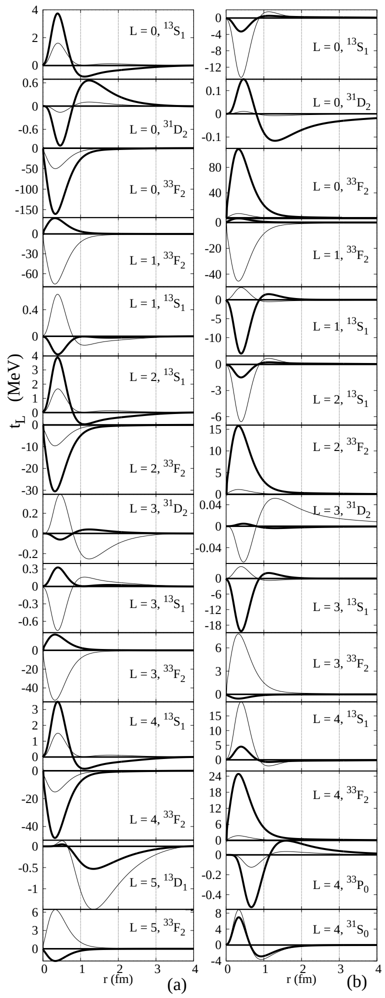

FIG. 3: $t_L(r)$ vs r for various $^{(2T+1)(2S+1)}\ell_j$ states, Real $t_L(r)$ thick line, Imaginary $t_L(r)$ thin line, at E_{Lab}= (a) 200 MeV and (b) 30 MeV.

(2) All $t_L^{S,T,E}(r)$'s vanish at r=0.
(3) For singlet, all the $t_L^{S,T,E}(r)$'s also vanish at r where $V(r)$ vanishes.
(4) At 30 MeV contributions from $^{13}S_1$ dominate while at 200 MeV the $^{33}F_2$ dominate.
(5) Expectations are that at in-between energies $^{33}P_2$ and $^{13}D_1$ will be dominating.

From these it is clear that compared to the M3Y form the proper t-MEI's are much different in shape (t(r→0)→0), multipole character as well as magnitude. In contrast to what has been available in the literature as M3Y form, which peaks at r=0, there is only L=0 contribution and the magnitudes are large. Moreover it contains central, tensor and $\vec{L}\cdot\vec{S}$ contributions quite similar to the first Born Approximation results. Besides this there are exchange contributions which are parametrized separately[10]. In our results all these are built in. Our formalism is general, non-pertrubative and provides results which can be incorporated in the FR-DWIA formalism directly without the bother of separate exchange contributions. One can use various realistic N-N interactions such as Paris, C. D. Bonn, Argonne V-18, etc and choose one unique interaction which fits both p-p elastic scattering as well as (p,2p) reactions at various energies, The FR-DWIA formalism incorporating these $t_L(r)$'s may also by useful to predict the (p, pn) and (n, 2n) cross sections for Accelerator Driven Systems, (ADS) reactors conceptualized for energy and neutron multiplication.

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