Influence of Effective Potential on the tuneling of the composite particle in the Coulomb Field

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Introduction

The quantum tunneling effect of a composite particle, which has intrinsic states, needs detailed investigation as experimental data [1] indicates that the penetration probability for loosely bound systems at low energies may be significantly more, contrary to conventional estimates. The problems involving tunneling and reflection of composite particles have been discussed using different models [2–8]. In this study, a deuteron is considered to approach from a long distance towards another deuteron, so that the study reduces to solving the problem of tunneling where one deuteron approaches in the Coulomb field of the other deuteron. "Deuteron" model is assumed for the composite particle where the system contains one neutral particle and one charged particle.

The Model

we consider one dimensional motion of the composite system containing one neutral and one charged particle having masses m_x and m_y coupled by the interaction $V(x-y)$ and slowly moving in the external potential U. The Hamiltonian of the system as

$$H = \frac{p_x^2}{2m_x} + \frac{p_y^2}{2m_y} + V(x-y) + U_x(x) + U_y(y), \tag{1}$$

where, we consider particles are situated at positions x and y and the external potential U acts on the two particles differently. Transforming the Eq. (1) in the center of mass coordinates R of the composite system while the relative coordinate of the constituent particles by r, one may construct the stationary Schrödinger equation as

$$\left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial R^2} - \frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial r^2} + V(r) + U_x\left(R + \frac{rm_y}{M}\right) + U_y\left(R - \frac{rm_x}{M}\right)\right]\Psi(R,r) = E\Psi(R,r), \tag{2}$$

where, M is the total mass of the two constituent particles in the composite system while μ is the reduced mass of the system in the center of mass coordinate system, E being the energy of the incident composite particle.

Therefore, the model essentially reduces to a problem comprising of a potential well representing the composite system moving in the coulomb field. We are interested to study evolution of intrinsic wave function of slow composite particle in the field of Coulomb potential for which one may consider an adiabetic ansatz at low energy like

$$\Psi(r,R) = \psi(R)\phi(r;R), \tag{3}$$

such that $\phi(r,R)$ is internal function describing the wave function inside the potential well while R is global variable representing the movement of the potential well. Following [8], we obtain $\psi(R)$ and $\phi(r;R)$.

The essential point is that new function $u(R)$ defined as Eq. (4)

$$\psi(R) = u(R)e^{-\int \alpha(R)dR}, \tag{4}$$

satisfies Eq. (5)

$$\frac{\partial^2 u}{\partial R^2} + \frac{2M}{\hbar^2}\left[E - \tilde{U}(R)\right]u = 0, \tag{5}$$

with effective potential $\tilde{U}$ as Eq. (6).

$$\tilde{U}(R) = \epsilon(R) - E_b + \frac{\hbar^2}{2M}\left[\alpha^2(R) + \frac{d\alpha(R)}{dR} - \beta(R)\right] \tag{6}$$

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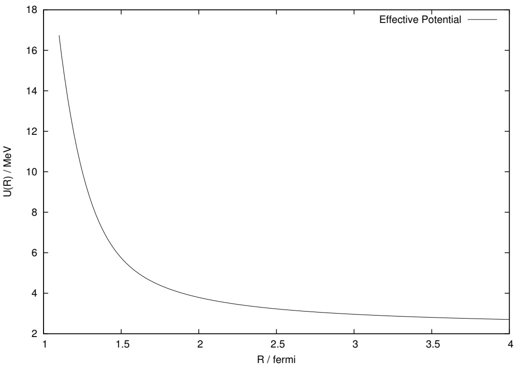

FIG. 1: The effective potential observed by the composite particle during quantum tunneling through Coulomb barrier.

where, we may define $\left\langle \phi \middle| \phi \right\rangle = N(R)$, $\left\langle \phi \middle| \frac{\partial \phi}{\partial R} \right\rangle = N(R)\alpha(R)$ and $\left\langle \phi \middle| \frac{\partial^2 \phi}{\partial R^2} \right\rangle = N(R)\beta(R)$ and E_b is the intrinsic binding energy of the constituent particles in the composite system. .

For our application, we consider "Deuteron" model containing one proton and one neutron while $U_x = U_p$ due to charge of proton while $U_y = 0$ due to neutron which is not influenced by the barrier.

Analysis

Eq. (6) is determined by solving Eq. (5) using Numerov algorithm. The effective potential is plotted in the Fig. 1. The wave function Ψ is determined for different incident energy of the particles. Figure 2 is plotted for the amplitude of the wave function Ψ against R for different incident energies E. This clearly indicates more probability of existence of particles toward the barrier and maximizes at a certain R after which the probability drops. The drop is not zero while smaller than the peak value indicating tunneling. Also it is observed that at lower energy values probablity of particles is more. Typical calculation indicates, say at $R = 1.1$ fm, $|\Psi^2(E = 0.01MeV)|/|\Psi^2(E = 1.0MeV)|$ is about 1.22 showing lower incident energy could be more favorable. This suggests that

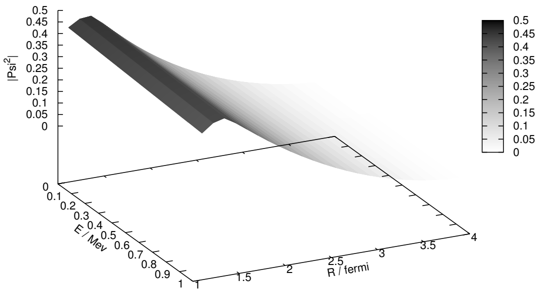

FIG. 2: The wave function $|\Psi|^2$ is plotted against R for different E.

fusion probablity could be more for a system with ground state intrinsic degenerate state [9].

References

[1] Arzhannikov, A V and Kezerashvili, G Ya, 1991, Phys. Lett. A, **156**, 514
[2] Saito N and Kayanuma Y, 1995, Phys. Rev. B, **51**, 5453
[3] Sato T and Kayanum Y, 2002, Europhys. Lett, **60**, 331
[4] Kimura S and Takigawa N, 2002, Phys. Rev C, **66**, 024603
[5] Goodvin G L and Shegelski M R, 2005, Phys Rev A, **71**, 032719
[6] Flambaum V V and Zelevinsky V G, 2005, J. Phys. G.: Nucl. Part. Phys., **31**, 355
[7] Bacca S and Feldmeier H, 2006, Phys Rev C, **73**, 054608
[8] Bertulani C A, Flambaum V V and Zelevinsky V G, 2007, J. Phys. G: Nucl. Part. Phys., **34**, 2289
[9] Jain S R, 2004, J. Phys. G: Nucl. Part Phys., **30**, 157