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Band Head Spin and Moment of Inertia for Super deformed Rotational Bands Using the VMI Model

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Introduction

Nearly all the super-deformed bands (SD) have been observed with hanging transitions; as a result only the γ-ray energies are known with no firm spin parity assignments [1]. Knowledge of the spin-parities is very crucial for their configuration assignments and for complete theoretical understanding of the SD bands. We have tested a very efficient method to assign spin-parities based on the Variable Moment of Inertia model (VMI) [2].

The VMI model [3] was first proposed by Mariscotti *et.al.* in 1969 to predict the energy levels of the ground state bands in $\mathcal{J}$ even-even nuclei. This model has later been used in many ways. Goel et.al [4] extended the VMI model to odd-odd nuclei and used it to predict the K-values of rotational bands by plotting the Mallmann's graph. We have now applied the VMI model to calculate the lowest spin (and hence the K-value) and also the gamma energies for the SD bands. The calculated band head spin is compared with other theoretical estimates available in the literature [5,7-15].

Method

From the VMI equations, the band head spin (I_o) and the moment of inertia ($\mathcal{J}_I$) is calculated by using the known gamma energy ratio (R), band head moment of inertia ($\mathcal{J}_0$) and softness parameter (σ) [2]. The two parameters $\mathcal{J}_0$ and C (or σ) is calculated by means of a least square fitting procedure considering all the experimentally known transition energies for SD band. Estimation of band head and other spins are compared with γ energies (Spin values were considered as I_0, I_0+2, I_0+4 etc). Obtained band head spin I_0 is then compared with other theoretical calculations. The results are very much in agreement. To verify estimated band head spin we have also plotted the ratio of transition energy over spin (RTEOS) versus spin as shown in Fig 1. The horizontal line is obtained for the "true" band head spin while hyperbola is obtained when higher or lower values is assigned to the actual band head spin.

Formalism

The VMI model equations gives the level energies of rotational bands as [3,4]

$$E_I(\mathcal{J}) = \frac{1}{2}C(\mathcal{J}-\mathcal{J}_0)^2 + [I(I+1)/2\mathcal{J}] \;\ldots\ldots\ldots\; (1)$$

Along with the equilibrium condition,

$$\partial E(\mathcal{J})/\partial \mathcal{J} = \mathbf{0} \;\ldots\ldots\ldots\; (2)$$

It determines the moment of inertia $\mathcal{J}_I$ for each state with spin I. $\mathcal{J}_0$ is a parameter defined as the ground state moment of inertia and C is the restoring force constant related to the softness parameter (σ). From equation (1) and (2) we get the cubic equation,

$$\mathcal{J}_I^{\,3} - \mathcal{J}_I^{\,2}\mathcal{J}_0 - [I(I+1)/2C] = 0 \;\ldots\ldots\ldots\; (3)$$

This cubic equation has one real root for any finite positive value of $\mathcal{J}_0$ and C. In SD rotational bands the E2 transition γ-ray energy is

$$E\gamma\,(I) = E\,(I) - E\,(I\text{-}2) \;\ldots\ldots\ldots\; (4)$$

Comparing equation (1) and (3), we get

$$E(I) = [I(I+1)/2\mathcal{J}_I]\{1 + [I(I+1)/4C\mathcal{J}_I^{\,3}]\}$$

$$E\,(I\text{-}2) = [(I-2)(I-1)/2\mathcal{J}_I]\{1 + [(I-2)(I-1)/4C\mathcal{J}_I^{\,3}]\}$$

These equations involve two parameters $\mathcal{J}_0$ and C. These two parameters characterize each nucleus defining the moment of inertia $\mathcal{J}_I$ and the transition gamma ray energies of the states of the bands. Gamma ray energies were well fitted from Eq. (4).

Results

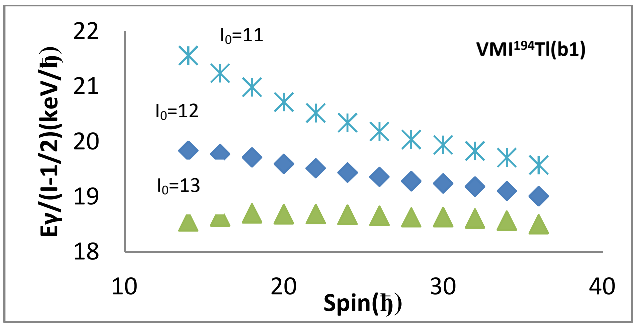
The results in general are in very good agreement between the calculated transitions energies from the VMI equation and the experimental values. The band head spin assignment for Hg, Tl and Pb of A=190 region nuclei using VMI model and values of associated parameters are given in Table I. These values are in agreement with the spin values reported in the literature, suggesting that our method works pretty well. The assignment of band head spins and the K-values is now being attempted in other SD bands also.

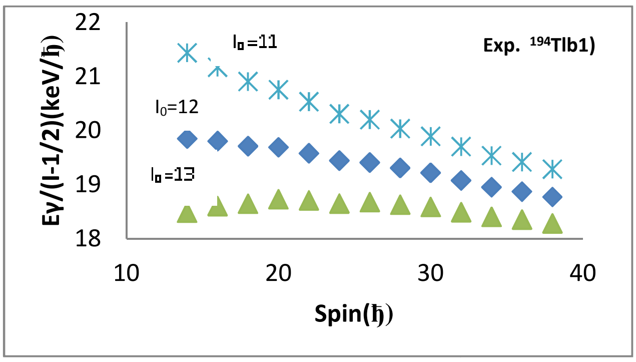
Table 1: The calculated band head spin by VMI equation for A=190 region. Comparison of band head spin from different methods, as available in literature

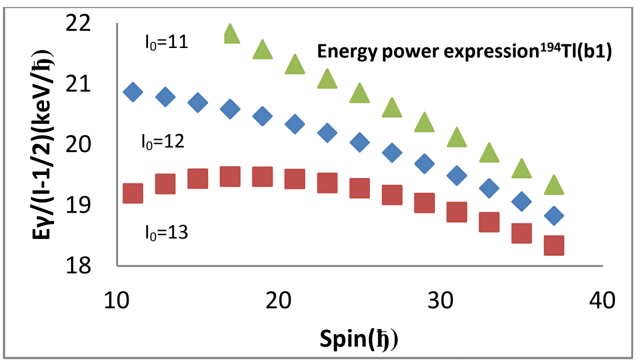
SD Band	Eγ Exp	Band head Energy (VMI)	R	σ x 10^{-5}	$\mathfrak{J}_o$	Presently assigned I_o (VMI)	Zhu & Shi[1]	ab[2]	Becker [3]	CBM[4]	SAM[5]	Wu& Zeng[6]
^{194}Tl (b1)	268	270	1.12	3.9	0.0998	12	12	12	12	12	-	12
^{194}Tl (b2)	220	221	1.16	3.9	0.0999	9	9	9	9	9	-	10
^{194}Tl (b3)	188	188	1.14	7.5	0.0948	10	10	10	10/11	10	-	8
^{194}Tl (b4)	207	207	1.15	7.5	0.0949	9	9	9	9/10	9	-	9
^{194}Tl (b5)	209	210	1.16	8.0	0.1007	8	8	8	8/9	8	-	9
^{194}Tl (b6)	241	242	1.15	4.0	0.1010	9	9	9	9/10	9	-	9
^{190}Hg (b1)	317	323	1.12	0.1	0.0835	12	13	13	12	12	12	-
^{190}Hg (b3)	279	278	1.11	4.4	0.1116	12	12	12	-	-	-	-
^{192}Hg (b1)	214	212	1.16	1.0	0.0892	8	8	8	8	8	8	8
^{192}Hg (b2)	241	243	1.14	7.5	0.0943	10	10	10	-	-	-	-
^{195}Pb (b1)	182	182	1.18	3.9	0.0987	7.5	7.5	-	-	7.5	7.5	-
^{195}Pb (b2)	163	163	1.20	3.9	0.0980	6.5	6.5	-	-	6.5	6.5	-
^{195}Pb (b3)	198	198	1.17	0.1	0.0907	7.5	7.5	-	-	7.5	7.5	-
^{195}Pb (b4)	214	216	1.15	7.3	0.0924	8.5	8.5	-	-	7.5	7.5	-

[1]Taken from Ref. [5], [2] Taken from Ref. [7-8],[3] Taken from Ref. [9], [4] Taken from Ref.[10,11], [5] Taken from Ref.[12,13,14],[6]Taken from Ref.[15] .

Fig 1: The VMI energies RTEOS verses spin plots are shown. The calculated VMI results are compared with experimental and energy power expression results.







References

[1] B.Singh, R.Zywina and R.B.Firestone C.S Wu, High Energy Phys. and Nucl.Phys.23 (1999) 1209,Nuclear Data Sheets 97 (2002) 241.

[2] Uma.V.S, Goel.A, and A.K.Jain, Proceedings of the DAE Symposium on Nuclear Physics 57 (2012) 361.

[3] Mariscotti, M.A.J, G.Scharff-Goldhaber,and B. Buck,Phys. Rev.C178 (1969) 1864.

[4] Goel. A and A.K. Jain, Modern Phys. Lett. A29(1990) 2403; Ph.D Thesis, I.I.T Roorkee (1992).

[5] MU Liang-Zhu and WU Chong-Shi, Commun. Theort. Phys.C43 (2005)719.

[6] X.L. Han and C.L.Wu, At. Data Nucl. Data Tables 73 (1999) 43.

[7] C.S.Wu,HighEnergyPhys.Nucl.Phys.(in Chinese)22(1998)71.

[8] C.SWu,HighEnergyPhys.andNucl.Phys.(inChinese)23(1999) 1209.

[9] J.A.Becker ,et.al,Phys.Rev.C46(1992)889

[10] Fu-Rong,and Ji-Min Hu, Phys.Rev.49(1994)1449.

[11] S.G.Zhou,C.K.Zheng,F.R.Xu,andJ.M.Hu,Nucl.Phys.A615 (1997) 229.

[12]Yu-XinLiu,Jian-GangSong,Hong-ZhouSun,andEn-GuangZhao,Phys.Rev.C56(1997)1370.

[13] Yu-XinL and Dong-FengGao, Phys.Rev.C63(2001)044317

[14] J.Tang and Y.X.Liu,HighEnergy Phys.Nucl.Phys.28 (2004).

[15] C.S.Wu, J.Y.Zeng, et.al,Phys. Rev.C45(1992) 261.

[16] NNDC (National Nuclear Data Centre). Brookhaven National Laboratory.[cited on June 2012] [Downloaded from http://www.nndc.bnl.gov/chart/]