Prolate dominant but oblate stable!

K.K.Girija[1*] and Antony Joseph[2†]
[1]*Department of Physics, M.P.M.M.S.N.Trusts College, Shoranur, Kerala, India-679122 and*
[2]*Department of Physics, University of Calicut, India-673635*

Introduction

The nuclear shapes, deformations and energy are interesting topics in the study of nuclear structure. Till date, no single theory is developed to describe the nuclear structure and properties completely. The repulsive Coulomb force between protons, the short ranged attractive nuclear force between nucleons, the shell effects and pairing correlation together decide the shape and deformation of nuclei. The nuclei exhibit spherical, quadrupole or higher order multipole deformed shapes. In nature, stable spherical nuclei are few in number [1]. Most of the nuclei are axially symmetric in the ground state (gs) and a prolate dominance over oblate shape is observed [2]. The coexistence of different shapes at the same spin and similar energies is also not rare [3],[4].

Since deformation affects nuclear potential energy (PE), the nuclei with different shapes and deformations attain equilibrium at different energies and the deformation corresponding to the minimum energy desides the shape of the nuclei in the ground state. Stable deformed nuclei with $\beta_2 < 0.3$ are commonly found in the rare earth region [5] and many attempts in the structural study of these nuclei are reported by various research groups [6],[7],[8].

This article is meant to discuss the yrast states of even-even nuclei in the isotopic chains $^{150-160}$Dy, $^{150-160}$Er and $^{150-160}$Yb and thereby to study the shape and deformation dependence of ground state energy, exploring cranked Nilsson Strutinsky shell correction method [9] and potential energy surface (PES) diagrams.

Theoretical formalisms

The nuclear shape and deformation are characterized by two collective parameters, the deformation parameter β or ε and the triaxiality γ. The rotational spectrum of a nucleus is represented by [10]:

$$E_I = \frac{\hbar^2}{2\jmath} I(I+1) \qquad (1)$$

where $\jmath$ is the moment of inertia and I is the total spin.

We have performed the total energy calculations for the even-even nuclei of the isotopic chains $^{150-160}$Dy, $^{150-160}$Er and $^{150-160}$Yb in the ground state. The range of triaxiality parameter used is 0-60^o and a frequency range of 0-1.5 MeV is employed.The quadrupole deformation parameter ε_2 is varied from 0 to 1.

Results and discussion

The PES diagrams of even-even $^{150-160}$Dy, $^{150-160}$Er and $^{150-160}$Yb nuclei for the ground state are drawn and one typical case is illustrated in Fig. 1. The quadrupole deformations and shapes corresponding to the equilibrium states are identified which are exhibited in Table 1. The minimum potential energies are also shown.

It is found that ^{150}Dy, $^{150-154}$Er and $^{150-156}$Yb are stable in the ground state with oblate shapes and the equilibrium energies predicted are 1.1 MeV, -1.4 MeV, 1.1 MeV, 2.1 MeV, -0.026 MeV , -0.2 MeV, -0.026 MeV and 0.091 MeV respectively. $^{152-160}$Dy are having prolate yrasts in the ground state with equilibrium energy 2.1 MeV, and ^{156,160}Er are stable in prolate shape with energies 2.1 MeV and 3.2 MeV respectively. ^{158}Yb has a triaxial minimum in the ground state at 2.0 MeV. In

*Electronic address: girijacus@gmail.com
†Electronic address: ajvar@rediffmail.com

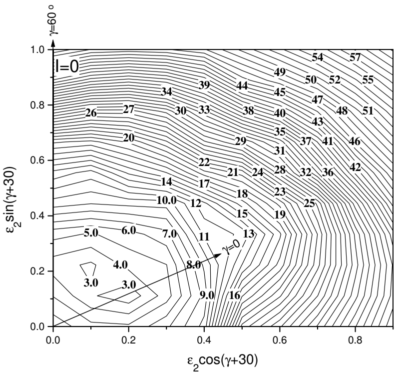

FIG. 1: PES diagram of ^{160}Yb isotope in the ground state.

the case of ^{158}Er and ^{160}Yb, there is the coexistence of a prolate minimum and a triaxial minimum in the ground state at energies 3.2 MeV and 3.0 MeV respectively.

The most striking point in the study is that stable oblate shapes are having less energy in comparison with the stable prolate and triaxial shapes of neighbouring nuclei in an isotopic chain.

In summary, in rare earth nuclei ground state potential energy is highly shape and deformation dependent. Even though majority of the stable nuclei in nature are prolate, yrast oblate minima are deeper than the yrast prolate and triaxial minima in an isotopic chain i.e., oblate shapes are more stable than prolate and triaxial shapes.

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TABLE I: The predicted ground state quadrupole deformations, shapes and energies.

Nucleus	gs shape	gs β_2	gs energy (MeV)
^{150}Dy	Oblate	-0.104	1.1
^{152}Dy	Prolate	0.201	2.1
^{154}Dy	prolate	0.237	2.1
^{156}Dy	prolate	0.260	2.1
^{158}Dy	prolate	0.274	2.1
^{160}Dy	Prolate	0.277	2.1
^{150}Er	Oblate	-0.094	-1.4
^{152}Er	Oblate	-0.106	1.1
^{154}Er	Nearly Oblate	0.168	2.1
^{156}Er	Prolate	0.214	2.1
^{158}Er	prolate Triaxial	0.248 0.254	3.2 3.2
^{160}Er	Prolate	0.267	3.2
^{150}Yb	Oblate	-0.145	-0.026
^{152}Yb	Oblate	-0.081	-2.0
^{154}Yb	Oblate	-0.100	-0.026
^{156}Yb	Oblate	0.150	0.91
^{158}Yb	Triaxial	0.187	2.0
^{160}Yb	Prolate Triaxial	0.225 0.236	3.0 3.0

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