Comparison of ground state Λ-binding energies of light hypernuclei

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Introduction

Earlier, we had obtained a semi-empirical formula [1] for ground state Λ-binding energies (B_Λ) of light hypernuclei, using an exponential form of Λ-nucleus potential, which gives a fairly good account of B_Λ data. Although, the ground state B_Λ of light hypernuclei have been satisfactorily explained [2,3] using Λ-nucleus potential in the folding model with a suitable density-dependent effective Λ-nucleon (ΛN) interaction, but these calculations have been done numerically. Hence, in order to obtain a broad insight into some of the important aspects, we do a comparative study of the ground state B_Λ of light hypernuclei obtained by us [1] and others [2-4].

Semi-empirical formula and some Λ-nucleus potentials

We solve the radial Schrödinger equation with the exponential form of the Λ-nucleus potential [1]:

$$V(r) = -V_0 e^{-r/a} ,$$

where $a = r'_0 A_c^{1/3}$.

Here, A_c is the core mass number of the hypernuclei and r'_0 is taken as a free parameter. Further simplifying, we get a semi-empirical formula for the Λ-binding energy in bound s-states of light hypernuclei [1]:

$$B_\Lambda = \frac{\hbar^2 A_c^{-2/3}}{8m_\Lambda r_0'^2\left((\pi^2/2)-1\right)^2}\Bigg\{(\xi_1+\xi_2) + \left[(\xi_1+\xi_2)^2 - (\pi^2-2)\left(\frac{\xi_1^2}{\pi^2} + \left(\frac{1}{2}-2n\right)\xi_2 + \frac{1}{8}\right)\right]^{1/2}\Bigg\}^2 + \frac{\beta' Z}{r_{rms}^3}, \quad (1)$$

where $\xi_1 = \frac{\pi^2}{2}\left((1/2)-2n\right)$ and

$$\xi_2 = \frac{\pi r'_0 A_c^{1/3}}{2}\left[\frac{8m_\Lambda V_0}{\hbar^2}\right]^{1/2}.$$

This semi-empirical formula (eq. 1), gives a fairly good account of the ground state B_Λ of light hypernuclei, if very light nuclei are ignored. These B_Λ values are shown as 1 in Fig. 1. The B_Λ of ${}^3_\Lambda H$, ${}^4_\Lambda H$ and ${}^4_\Lambda He$ are the predicted values. We have also plotted B_Λ values [1] obtained from χ^2- fitting using eq.(1) in which the last term is replaced by $\beta'\left(\frac{Z}{A_c}\right)$. The results are quite reasonable when very light hypernuclei, ${}^3_\Lambda H$, ${}^4_\Lambda H$ and ${}^4_\Lambda He$ and the known 'troublesome' nuclei, ${}^7_\Lambda Be$ and ${}^9_\Lambda Be$, are excluded from the fit. The B_Λ of these excluded nuclei are the predicted values. These fitted and the predicted values are represented as 2 in Fig. 1.

We also carry out exact numerical fitting of ground state B_Λ data of light hypernuclei with core mass number ranging from 2 to 15, using exponential form of Λ-nucleus potential. The calculated B_Λ for hypernuclei with A_c = 3, 4 and 6 are the predicted values. These results are shown as 3 in Fig. 1.

Ahmad et al. [2], have calculated the B_Λ values for light hypernuclei in the folding model with a density-dependent effective ΛN interaction. The Λ-nucleus potential [2] is

$$V_\Lambda(r) \approx \overline{V}_0 \tilde{\rho}(r)[1-\beta\rho^{2/3}(r)] ,$$

where $\tilde{\rho}(r)$ is assumed to be the same as the charge distribution of the core nucleus. $\overline{V}_0$ and β are treated as parameters. The numerically calculated B_Λ values, plotted as 4 in Fig. 1, correspond to their potential parameter set B [2].

Mahmood Mian [3] using a three-parameter density-dependent ΛN force in the folding model, obtained the Λ-nucleus potential as

$$V_\Lambda(r) = \frac{\overline{V}_0}{2\pi^2}\int F(q)\exp\left[-\frac{q^2 d^2}{4}\right] q^2 j_0(qr)dr ,$$

where the symbols have their usual meaning. The numerically calculated B_Λ values [3] in the ground state of light hypernuclei, corresponding to the potential parameter set A [3], are plotted as 5 in Fig. 1. The average size parameter of the nuclear density for N ≠ Z core hypernuclei was also determined [3] by numerically fitting their experimental B_Λ data. The theoretical B_Λ values so obtained [3] are plotted as 6 in Fig. 1.

Employing a free-state two-body ΛN gaussian potential along with a δ-function three-body ΛNN force, the χ^2-fit to the experimental B_Λ data was performed by Ansari et al. [4]. The corresponding B_Λ values obtained for their set AII [4] are plotted as 7 in Fig. 1.

Result and Discussion

The calculated ground state B_Λ values of light hypernuclei are plotted versus A_c in Fig. 1, along with the available experimental B_Λ data. As the error bar in the experimental B_Λ data of ${}^{16}_{\Lambda}O$ is not available, we take a plausible value of 5% of the experimental B_Λ as the error in the datum. The B_Λ values calculated from our semi-empirical formula (eq. 1) as well as from the numerical fitting using an exponential potential, give a fairly good agreement with the experimental B_Λ data. Predicted B_Λ values, though slightly lower for ${}^{4}_{\Lambda}H$ and ${}^{4}_{\Lambda}He$ are also qualitatively acceptable. When the last term of B_Λ formula (eq. 1) is expressed as $\beta'\left(\frac{Z}{A_c}\right)$, the formula distinguishes between nuclei with different Z but same A_c, though the size effect is ignored. The results, however, are more or less similar except for some differences at higher A_c.

The B_Λ values of Ahmad et al. [2], where $\tilde{\rho}(r)$ was assumed to be the same as the charge distribution of the core nucleus, are represented as 4 in Fig. 1. The calculated B_Λ values for A_c = 6, 7, 15 and ${}^{9}_{\Lambda}B$, ${}^{10}_{\Lambda}B$, ${}^{12}_{\Lambda}C$ are obtained with potential parameter set B as given in ref. [2]. The available ground state B_Λ is satisfactorily reproduced with slight difference in few cases, especially beyond A_c = 11.

The B_Λ values obtained by Mahmood Mian [3], taking into account the range of Λ-nucleon interaction explicitly, referred to as 5 in Fig. 1, show deviation at higher A_c (>12). Another set of B_Λ values determined [3], to obtain average size parameter of the nuclear density for N ≠ Z core hypernuclei, referred to as 6 in Fig. 1, are somewhat on the lower side of the experimental data at higher A_c.

Ansari et al. [4] calculated B_Λ values from χ^2- fit to the experimental B_Λ data of hypernuclei with core mass number ranging from 9 to 14. Their results, represented as 7 in Fig. 1, show slight over binding for A_c = 13 and 14.

The results obtained in refs. [2-4] give a reasonable account of the data, except for some deviations at higher A_c. Fairly satisfactory reproduction of ground state B_Λ of light hypernuclei obtained by our semi-empirical formula [1] and from our numerical fitting of the B_Λ data indicates that even an exponential Λ-nucleus potential is quite adequate in reproducing B_Λ of many light hypernuclei. Moreover, it leads to a semi-empirical formula which gives fairly good results. However, more work is needed to analyze whether any deeper significance can be ascribed to the success of exponential potential.

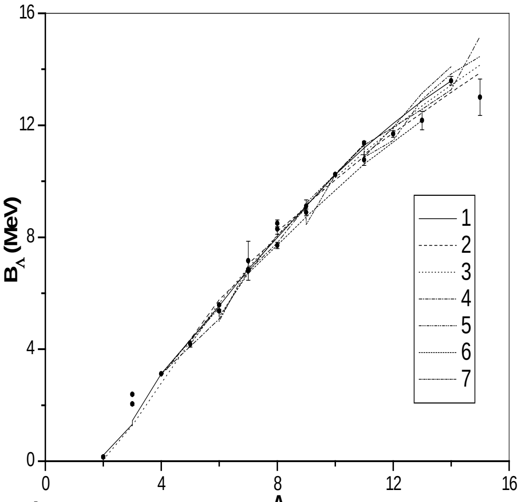

Fig. 1

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