Progress towards a relativistic breakup reaction theory

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Introduction

Breakup reactions of nuclei (especially those near the drip lines) in the Coulomb and nuclear fields of the target nuclei have important applications in understanding the properties of exotic nuclei. Theories of breakup reactions like the distorted wave Born approximation (DWBA) and the Continuum discretized coupled channels (CDCC) can be constructed by suitable choice of the exact scattering wave function in a post or prior form reaction theory. What is interesting however, is that many of the recent data are reasonably well explained by these methods as also by elaborate models like the dynamical eikonal approximation and the time dependent Schrödinger equation method. This is surprising because all these models are based on different approximations. An obvious explanation, though, is that most of the reaction observables calculated - like the momentum distributions, relative energy spectra, angular distributions- are of an inclusive nature and involve a lot of summations or integrations, which suppress the contribution of many partial waves. There is, thus, a strong case to calculate more exclusive observables like the double- and triple-differential cross sections, where the differences within the theories could be more apparent.

Also interesting is the beam energy at which breakup reactions involving exotic nuclei are performed. Most of the first generation experimental facilities (except at GSI) used to operate at beam energies less than 100 MeV/u. In the recent upgrades, beam energies around 350 MeV/u (RIKEN) and 1 GeV/u (GSI) are common. So, if one is not satisfied in simply plugging in relativistic kinematics into the non relativistic Schrödinger equation, one could question the applicability of the Schrödinger equation itself for such beam energies. Some initial thoughts in this direction have already been proposed in Ref. [1].

So a relativistic description of the breakup process incorporating full Coulomb (and nuclear) interactions between the fragments and the target, with final state interaction between breakup fragments and target nucleus taken to all orders (three or four body final states) is a theory that one would be looking for. *Especially for the Coulomb part, one can investigate if analytic or semi-analytic expressions for the Coulomb amplitude can be derived, which could serve as benchmark for other theories. A coherent addition of the Coulomb and nuclear amplitudes will enable identification the Coulomb-nuclear interference effects.* To the best of our knowledge a systematic study of Coulomb-nuclear interference, especially in inelastic breakup, has not been investigated before.

In this contribution we discuss our efforts towards the construction of such a theory and the kinematical conditions one need to fulfill in these reactions to justify the application of the non-relativistic Schrödinger equation.

Formalism and Results

We aim a beam of projectile a on a target t and consider completely elastic scattering with a breakup reaction of the form ($a + t \rightarrow b + c + t$) in the Coulomb field of the target. Working in the centre of mass (CM) frame, we try, first, to investigate which reaction observables can be retained at higher, relativistic beam energies using the non-relativistic Schrödinger approach. Although the beam energy may be high, the relative energy between the fragments in teh final channel maybe small. Indeed, it is known that for systems of exotic nuclei (viz., ^{11}Be or ^{19}C), the relative

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energy peak occurs at low energies [2]. Then, the relative energy cross section is given as:

$$\frac{d\sigma}{dE_{rel}} = \int_{\Omega_{at}\Omega_{bc}} d\Omega_{at} d\Omega_{at} \sum_{l,m} |\beta_{lm}|^2 \frac{2\pi}{\hat{l}\hbar v_{at}} \frac{\mu_{at}\mu_{bc}p_{at}p_{bc}}{h^6} \quad (1)$$

where, $\hat{l}$ is $\sqrt{2l+1}$, E_{rel} is the *b*-*c* relative energy in the final channel, v_{at} is the *a*-*t* relative velocity in the initial channel, Ω's are the solid angles, μ's are the reduced masses and p's are the linear momenta of *b*-*c* and *a*-*t* systems respectively. The reduced amplitude β_{lm} [2] is given in the finite range distorted wave Born approximation (FRDWBA) by

$$\beta_{lm}^{FRDWBA} = \int\int d\mathbf{r}_1 d\mathbf{r}_i \chi_b^{(-)*}(\mathbf{q}_b, \mathbf{r}) \chi_c^{(-)*}(\mathbf{q}_c, \mathbf{r}_c) V_{bc}(\mathbf{r}_1) \phi_a^{lm}(\mathbf{r}_1) \chi_a^{(+)}(\mathbf{q}_a, \mathbf{r}_i). \quad (2)$$

The χ's are the pure Coulomb distorted waves for the respective particles with (+) showing the outgoing wave boundary condition and ingoing for (-). $\phi_a^{lm}(\mathbf{r}_1)$ is the ground state wavefunction of a with angular momentum l and projection m. The β_{lm} is a complex integral which can be factorized into a structure part and an analytic dynamics part (in terms of the Bremsstrahlung integral).

As a start, we did the relativistic and non-relativistic calculations for the kinematical quantities and plot here the magnitude of the wave vector q_b (i.e., k_b in the CM frame for the projectile fragment *b*) as a function of the total beam energy E_a for a fixed value of q_c (wave vector of *c*) for the breakup of ^{11}Be on ^{208}Pb. The results are in shown in fig. 1. We see that there is a very good correlation between the non-relativistic and the relativistic curves at lower beam energies. This is to be expected since the relativistic calculations should ultimately break down to non-relativistic calculations at the low energy limit. Nevertheless, for a beam energy of more than 1 GeV/u for ^{11}Be, the difference in the two curves is more pronounced. Such a difference in the wave vector could lead to a large difference in the actual energy distributions and thus reflect significant variations in the calculations for the various reaction observables.

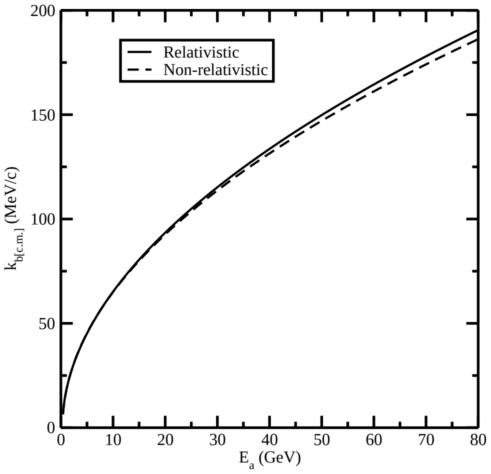

FIG. 1: Plot of magnitude of q_b (i.e., k_b in the CM frame) vs total beam energy E_a for a fixed value of q_c (E_c = 300MeV). The non-relativistic calculation is represented by the dashed line while the solid curve shows its relativistic counterpart.

However, since the relative energy can be small even though the individual energies of the final fragments are large, the Schrödinger equation mechanics (applicable for the non-relativistic regime) can be applied there as well. We shall investigate this feature for the breakup of ^{11}Be and ^{19}C in the field of a heavy target at high beam energies.

Acknowledgments

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References

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