Strong decays of radially excited equal mass vector mesons

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1. Introduction: In this work, we have studied strong decays of first radial excitations, ω(1420) and ρ(1450) (regarded as 2^3S_1 excitations) of lightest vector mesons, ω(782) and ρ(770) through the process V'→V+P (with V' and V being the radially excited and ground state vector mesons respectively, while P being the ground state pseudoscalar meson), which proceeds through the quark-triangle loop diagrams in the framework of Bethe-Salpeter Equation (BSE) under Covariant Instantaneous Ansatz (CIA)[1]. The decay widths for the processes ω(1420) →ρ(770) + π(140), and ρ(1450) →ω(782)+π(140) in the above framework are in rough agreement with data. These hadronic transition amplitudes which involve quark triangle loops represent sensitive probes into the internal dynamics of $q\bar{q}$ hadrons. and are particularly convenient for testing finer details of any model for strong interaction dynamics. They are evaluated through hadron-quark vertex functions $\Gamma(\hat{q})$ operative at the hadron-quark vertices in the Feynman diagrams for such processes. In this connection, we wish to mention that though a considerable amount of work has been done in decays of equal mass ground state vector mesons such as, $\rho \rightarrow \pi + \pi$, not much work has been done so far in decays of these radially excited vector mesons such as ω(1420) and ρ(1450). For some of the recent works on these mesons in various models, see Ref.[2]. It is seen that although there is general consensus on the existence of these states, there is considerable discrepancy on their decay widths. Also their experimental status is not very clear as can be seen from the recent PDG Tables [3], which shows a wide range of variation in data on their decay widths. Thus further investigations both in theory and experiment are required. The theoretical study of radially excited mesons is expected to provide us with a deeper understanding of internal structure of hadrons and of the effective inter-quark interactions.

These strong decays of excited vector mesons, ω(1420) and ρ(1450) are studied in the framework of Bethe-Salpeter Equation (BSE) under Covariant Instantaneous Ansatz (CIA) [1,4] , which is a Lorentz-invariant generalization of Instantaneous Approximation (IA). For a $q\bar{q}$ system, the CIA formulation ensures an exact interconnection between 3D and 4D BSE, where the 3D BSE serves for making contact with the 3D mass spectrum, while the 4D BSE provides the hadron-quark vertex which satisfies a 4D BSE with a natural off-shell extension over the entire 4D space and thus provides a Lorentz-invariant basis for evaluation of various transition amplitudes through various quark-loop diagrams. This framework has earlier given good predictions for a number of processes like: $V \rightarrow e^+ + e^-$, $P \rightarrow l + \nu_l$, $P \rightarrow \gamma + \gamma$, $V \rightarrow P + \gamma$, as well as the recent high energy tests of this model in the study of double charmonium production in electron-positron annihilation at energies $\sqrt{s}$= 10.6 GeV through the process, $e^- + e^+ \rightarrow J/\psi + \eta_c$ (see [4-8] and references therein).

The framework of Bethe-Salpeter Equation (BSE) that we have employed here is a conventional non-perturbative approach in dealing with relativistic bound state problems in QCD and is firmly established in the framework of Field Theory. And from the solutions we obtain useful information about the inner structure of hadrons which is also crucial in high energy hadronic scattering and production processes. Despite its drawback of having to input model dependent kernel, these studies have become an interesting topic in recent years since calculations have shown that BSE framework using phenomenological potentials can give satisfactory results as more and more data is being accumulated. We get useful insight about the treatment of various processes using BSE due to the unambiguous definition of the 4D BS wave function which provides exact effective coupling vertex (Hadron-quark vertex) of the hadron with all its constituents (quarks). This Hadron-quark vertex is considered to sum up all the non-perturbative QCD effects in the hadron.

2. Strong decays of radially excited vector mesons: The leading order quark-triangle diagram for the process V'->V+P is shown in Fig.1 where the masses of mesons are M, M' and M'' respectively with four-momenta P, P' and P'' and the labeling of momenta of internal quark lines as shown below.

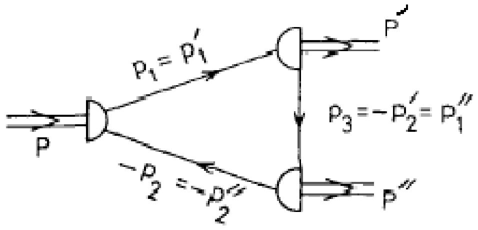

Fig.1: Leading order quark triangle diagram for the process, $V^{'} \to V + P$. Other diagram is obtained by reversing the arrows of internal quark lines.

The invariant amplitude for the process is [5]:

$$M_{fi} = \sqrt{\frac{2}{3}}(2\pi)^8 \int d^4\, q[TR\,], \qquad (1)$$
$$[TR] = Tr[\Gamma(\hat{q})S_F(p_1)\Gamma'(\hat{q}')S_F(p_3)\Gamma''(\hat{q}'')S_F(-p_2)],$$

with [TR] being the trace over the product of gamma matrices. Here, $\sqrt{2/3}$ is the color-isospin factor and $\Gamma(\hat{q})$, $\Gamma'(\hat{q}')$ and $\Gamma'(\hat{q}'')$ are the three hadron-quark vertex functions for the excited vector meson, V′, ground state vector meson, V and ground state pseudoscalar meson, P respectively with detailed structures given in Ref.[5] as:

$$\Gamma'(\hat{q}') = (i\gamma.\varepsilon')N'D'(\hat{q}')\varphi'(\hat{q}')/2\pi i, \qquad (2)$$
$$\Gamma''(\hat{q}'') = \gamma_5 N''D''(\hat{q}'')\varphi''(\hat{q}'')/2\pi i,$$
$$\Gamma(\hat{q}) = (i\gamma.\varepsilon)ND(\hat{q})\varphi(\hat{q})/2\pi i$$

respectively. Here ε and ε' are the polarization vectors of the vector mesons V' and V respectively. N, N' and N'' are the 4D Bethe-Salpeter normalizers for the three mesons which are evaluated through current conservation condition. D, D' and D'' are the respective denominator functions for the three hadrons (which plays an important role in bringing out an exact interconnection between 3D and 4D BSE and thus helps in identifying the 4D hadron-quark vertex function for a hadron), while φ' and φ'' (in the final state of the process) are the corresponding ground state 3D BS wave functions for vector and pseudoscalar mesons with internal momenta q' and q'', which satisfy the corresponding 3D BSE (on surfaces P'.q'= 0 and P''.q''= 0) [1,4], that is appropriate for making contact with the 3D mass spectrum and is reducible to equation for a 3D harmonic oscillator with coefficients depending on the mass M and total quantum number N. The ground state wave functions deducible from this equation (for ground state hadrons V and P) have Gaussian structure in the harmonic oscillator basis and are expressed as: $\varphi'(\hat{q}') \approx Exp[-\frac{\hat{q}'^2}{2\beta'^2}]$ and $\varphi''(\hat{q}'') \approx Exp[-\frac{\hat{q}''^2}{2\beta''^2}]$, while that for the first radially excited vector meson V', the 3D BS wave function is guessed as $\varphi(\hat{q}) \approx (-3\beta^2 + 2\hat{q}^2)Exp[-\frac{\hat{q}^2}{2\beta^2}]$ using variational arguments. Here β' , β'' and β are the inverse range parameters which incorporate the content of BS dynamics. The invariant amplitude in Eq.(1) after trace evaluation is expressible as [5]:

$$M_{fi} = i4\sqrt{\frac{2}{3}}(2\pi)^5 mNN'N''\varepsilon_{\mu\rho\sigma\tau}\varepsilon_\mu\varepsilon'_\rho P_\sigma Q_\tau X, \qquad (3)$$

with P being the total momentum, while 2Q= P'-P'' being the difference in momenta of the two outgoing mesons. X is the four-dimensional integral over $d^4q = d^3\hat{q}Md\sigma$ of the product of three $[D(\hat{q})\varphi(\hat{q})]$ type terms corresponding to the three participating hadrons in the process which is difficult to solve and is worked out as in Ref.[5]. The total decay rate for the process is worked out as:

$$\Gamma = 16(2\pi)^9 m^2 N^2 N'^2 N''^2 |\vec{P}'|^3 |X|^2/81. \qquad (4)$$

The calculated values of decay widths along with central values of data in brackets are:

$\Gamma(\omega(1420) \to \rho + \pi) = 141MeV$ (Expt.=(125.8–174.7) MeV [3]),
$\Gamma(\rho(1450) \to \omega + \pi) = 98.6MeV$ (Expt.=84.0MeV [3])
and are in rough agreement.

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