# Melting of shell effects in Lead isotopes

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## Introduction

Semiclassical analysis of shell effects in the density of energy levels at finite temperature is presented by considering realistic nuclear potentials. The oscillating part of level density is attributed to the properties of classical periodic orbits by "trace formulae"[1]. A particularly nice manifestation of this connection is the occurrence of shell effects in the level density. Shell effects in the level density are modified dramatically with the inclusion of spin-orbit interactions to harmonic oscillator potentials [4].

Here we derive an exact trace formula for level density of spherically symmetric harmonic oscillator potential including spin-orbit interactions at finite temperatures ($g_T(E)$) and test it to determine the level density parameter $a(T)$ as a function of temperature in $^{206}Pb$, $^{207}Pb$ and doubly magic $^{208}Pb$ nuclei:

$$a(T) = \frac{\pi^2 g_T(\epsilon_F)}{6},$$

where $\epsilon_F$ is the chemical potential of the nucleus. A commonly used expression for level density $\rho(E^*)$ is given by Bethe's formula [2] in which the knowledge of so-called level density parameter is necessary. Recent experimental results [3] on the level density parameters of $Pb$ isotopes have rekindled our interest and led us to the present investigation.

Hamiltonian $\hat{H}$ for three dimensional harmonic oscillator potential including spin-orbit interactions is given as:

$$\hat{H} = \hat{H}_0.\mathbb{I} - \hbar\kappa\hat{\mathbf{C}}(\mathbf{r},\hat{\mathbf{p}}).\hat{\boldsymbol{\sigma}},$$
$$\hat{\mathbf{C}}(\mathbf{r},\hat{\mathbf{p}}) = \nabla\mathbf{V}(\mathbf{r}) \times \hat{\mathbf{p}}$$

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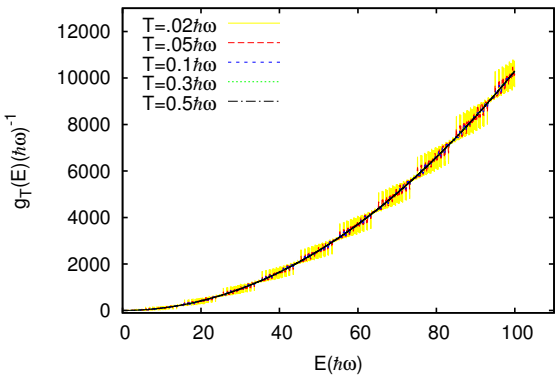


FIG. 1: $g_T(E)(\hbar\omega_0^{-1})$ vs. $E(\hbar\omega_0)$ at four different temperatures.

with

$$\hat{H}_0 = \frac{1}{2}\hat{\mathbf{p}}^2 + \frac{1}{2}\sum_{i=x,y,z}\omega_i^2 r_i^2$$

is the harmonic part of the Hamiltonian. $\hat{\mathbf{C}}(\mathbf{r},\mathbf{p})$ is an arbitrary vector function of coordinate $\hat{\mathbf{r}}$ and momentum operators $\hat{\mathbf{p}}$, $\hat{\boldsymbol{\sigma}} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. $\kappa$ is the spin-orbit interaction strength parameter measured in units of $(\hbar\omega_0)^{-1}$. The three oscillator frequencies are expressed in terms of deformation parameters $\alpha, \beta$ as given below:

$$\omega_x = \omega_0, \quad \omega_y = (1+\alpha)\omega_0, \quad \omega_z = (1+\alpha)^{\beta}\omega_0.$$

For $\alpha \neq 0, \beta = 1$ the system has axial symmetry and for $\alpha = 0, \beta = 1$ the system has spherical symmetry. Semiclassical trace formula producing level density $g(E)$ for spherically symmetric harmonic oscillator potentials with spin-orbit interactions has been developed by Amann et al. [4]

Canonical single-particle partition function at finite temperature $Z_T(\beta)$ can be obtained

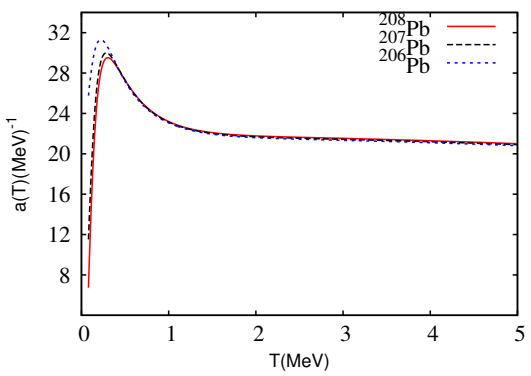


FIG. 2: Plot of level density parameter $a(T)$ vs. $T$ for $Pb$ isotopes.

after evaluating Laplace transform of the level density using the following relation [5]:

$$Z_T(\beta) = \mathcal{L}_\beta \left[g(E)\right] \frac{\pi\beta T}{\sin \pi\beta T}.$$

where $\beta$ is inversely proportional to temperature $T$ in quantum statistics and thermodynamics but in present context it is just a mathematical variable which could even be complex.

An explicit form of the single-particle level density at non-zero temperature can be found by taking an inverse Laplace Transform:

$$g_T(E) = \mathcal{L}_E^{-1} \left[Z_T(\beta)\right].$$

We plot in Fig. (1) the level density vs. energy at four different temperatures and find that the shell effects are smoothed out at higher temperatures.

Once the level density at finite temperature is known to us, we can determine the chemical potential ($\epsilon_F$) by the following relation:

$$N, Z = \int_0^{\epsilon_F} g_T(E) dE.$$

Spin-orbit interaction with strength paramter $\kappa$ is taken as 0.062 $\hbar\omega_{0n}^{-1}$ in the calculations involving neutron number and 0.065 $\hbar\omega_{0p}^{-1}$ while considering protons number [6] where

$$\hbar\omega_{0n,p} = \frac{41}{A^{1/3}} \left(1 \pm \frac{N-Z}{A}\right)^{1/3} MeV.$$

We plot the level density parameter $a(T)$ for $^{206}Pb$, $^{207}Pb$ and $^{208}Pb$ nuclei respectively, using the following relation:

$$a(T) = \frac{\pi^2 (g_T(\epsilon_F^N) + g_T(\epsilon_F^Z))}{6}$$

in Fig. (2).

We notice that the shell effects are very dominant in low temperature regime ($\leq 1MeV$) and in $T \geq 1MeV$ region the level density parameter ($\sim A/10MeV^{-1}$) is almost independent of temperature. $^{208}Pb$ is a doubly magic nucleus with closed shells and its level density parameter is much lower ($\sim A/29MeV^{-1}$) than other two isotopes at temperature $T = 0.08MeV$.

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