Multifractality in Multiparticle Production in 4.5 and 14.5A GeV/c ^{28}Si-AgBr Collisions

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Introduction

In dynamical systems, intermittency is an irregular alteration of phases of seemingly periodic and chaotic dynamics. Intermittent behaviour is commonly observed in fluid flows. However, intermittency was first observed in the cosmic ray event [2]. Thereafter, it was observed in hgh-energy hadron-hadron [3], hadron-nucleus [4] and nucleus-nucleus collisions [5]. Bialas and Peschanski [1] have proposed scaled factorial moments, Fq, to study intermittent behaviour of particles produced in relativistic nuclear collisions. Investigations have confirmed a power-law behaviour of scaled factorial moments: F_q α $(d\eta)^{-\varphi_q}$. Thus, intermittency seems to be an important feature of multiparticle production dynamics, which is also compatiable with prediction of cascade models. Self-similarity is closely linked to multifractality; multifractality is used to investigate fluctuations in multiplicity distributions of particles produced in relativistic hadronic and nuclear collisions. An useful formalism has also been developed by Chiu and Hwa [6] for investigating the occurrence of fractal structure in multiplicity distributions of particles produced in such collisions.

Chiu and Hwa have defined Gq moments and estimated the values of the parameters that characterize the fractal properties. The values of Gq moments, where q refers to order of the moments, are calculated for each event and then averaged over all the events considered for analysis. It may be noted that Gq moments unlike Fq moments exhibit self-similar behaviour in particle density distribution which lead to power-law behaviour: Gq α $(d\eta)^{-T_q}$ as $\delta\eta \to 0$, where $\delta\eta = \frac{\Delta\eta}{M}$, M is bin number, $\Delta\eta = \eta_{max} - \eta_{min}$ and η_{min} and η_{max} are the values of the minimum and maximum pseudorapidities in each event in the data sample The approach of Gq moments has inherent disadvantage because the variation of ln<Gq> with -lnδη , shows that it saturates as $\delta\eta \to 0$. Furthermore, it has been found that in low multplicity events, Gq moments are dominated by statistical fluctuations. In order to address these issues, Hwa and Pan [7] have redefined Gq moments by introducing a cerntain step function. The modified form of Gq moments overcomes the saturation effect and supresses the associated statistical fluctuations. An attempt is, therefore, made to investigate some interesting features of multifractality in terms of modified Gq moments in 4.5 and 14.5A GeV/c ^{28}Si-AgBr collisions.

Further, the results are compared with those obtained for the Monte Carlo AMPT generated events.

Expermental Details

In the present study two emulsion stacks exposed to 4.5 and 14.5A GeV/c Silicon beams at JINR (Dubna) and AGS (BNL) respectively are used. Random samples of 212 and 314 interactions due to AgBr are selected using standard emulsion method. Using similar criterion 600 Monte-Carlo AMPT generated events are also analyzed for the purpose of comparison.

Results and Discussion

Modified Gq moments are calculated using the following expression.

$$G_q = \sum_{m=1}^{M} \left(\frac{n_m}{N}\right)^q \theta(n_m - q) \qquad (1)$$

where q is a positive integer and n_m denotes the multiplicity of the particles in mth bin of width δη, where $N = \sum_{m=1}^{M} n_m$. The average of Gq moments may be calculated from

$$\langle Gq \rangle = \frac{1}{N_{evt}} \sum_{allevents} G_q \qquad (2)$$

Values of < Gq > for q = 2—6 are calculated using Eqs. 1 and 2. ln<G_q> versus lnM plots for 4.5 and 14.5A GeV/c ^{28}Si-AgBr collisions for the experimental and AMPT generated data are displayed in Figs 1 and 2. The values of ln<G_q> exhibit a linear dependence on lnM for both the data sets, indicating thereby a power-law behaviour of the form: <Gq> α $(d\eta)^{-T_q}$. This result also indicates that our data reveal self-similarity in the particle production process. Values of the slope parameters, T_q, are obtained by least squares fits to the data for both the experimental and AMPT generated data. Values of Tq for different order of the moments, q, are presented in Table 1. It is interesting to note from the table that slope parameter, T_q, increases with increasing order of the moments for the data on 4.5 and 14.5A GeV/c ^{28}Si-AgBr collisions.

The self-similar characteristics of fractals, characterized by generalized dimensions, D_q, are calculated by using the following relationship:

$$D_q = \frac{Tq}{q-1} \qquad (3)$$

Values of Dq for q=2—6 are calculated for 4.5 and 14.5A GeV/c ^{28}Si-AgBr collisions for both experimental and AMPT data are listed in Table 2. It may be interesting to note from the table that generalized dimensions, Dq, decrease with increasing order of the moments, which confirms the presence of multifractality in our data. However, it is observed to be essentially insensitive to projectile energy. The values of Dq are found to be practically the same for the experimental and AMPT generated data.

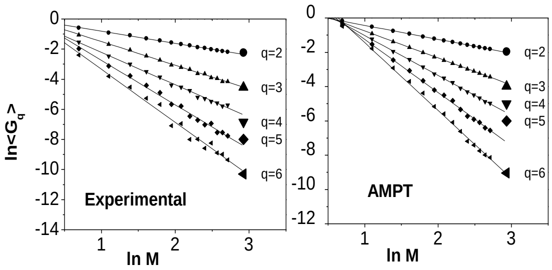

Fig. 1 Variations of ln<G_q> with ln M for 4.5A GeV/c ^{28}Si-AgBr collisions.

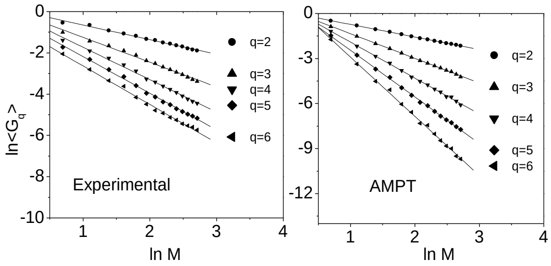

Fig. 2 Variations of ln<G_q> with ln M for 14.5A GeV/c ^{28}Si-AgBr collisions.

Conclusions

Analysis of the data on 4.5 and 14.5A GeV/c ^{28}Si-AgBr collisions leads to the following important conclusions:

1. The linear dependence of ln<Gq> on lnM reveals power-law behaviour.

2. The decrasing trend of Dq with q indicates presence of multifractality in the multiparticle production data, which is supported reasonably nicely by the AMPT model.

Table 1 Values of slope parameters, Tq, for q =2—6 for ^{28}Si-AgBr collisions.

Energy per nucleon (GeV)	Order of moments (q)	Slope parameter (T_q)	
		Experimental	**AMPT**
4.5	2	0.820±0.007	0.824±0.009
	3	1.572±0.022	1.578±0.004
	4	2.160±0.042	2.326±0.014
	5	2.952±0.066	3.101±0.038
	6	3.517±0.127	3.957±0.063
14.5	2	0.708±0.019	0.834±0.003
	3	1.202±0.030	1.641±0.012
	4	1.571±0.031	2.396±0.024
	5	1.778±0.028	3.105±0.025
	6	1.865±0.036	3.947±0.051

Table 2 Values of generalized dimensions, Dq, for q=2—6 for ^{28}Si-AgBr collisions.

Energy per nucleon (GeV)	Order of moments (q)	Generalised dimensions (D_q)	
		Experimental	**AMPT**
4.5	2	0.801±0.018	0.813±0.003
	3	0.756±0.028	0.789±0.012
	4	0.734±0.031	0.775±0.024
	5	0.724±0.033	0.761±0.036
	6	0.703±0.036	0.740±0.051
14.5	2	0.708±0.002	0.834±0.003
	3	0.601±0.003	0.820±0.005
	4	0.528±0.001	0.794±0.006
	5	0.444±0.005	0.785±0.013
	6	0.373±0.011	0.776±0.016

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