# The effect of medium on the relaxation of the dissipative flows in an interacting pion gas

Utsab Gangopadhyaya,[*] Sukanya Mitra,[†] and Sourav Sarkar[‡]
*Theoretical Physics Division, Variable Energy Cyclotron Centre,*
*1/AF Bidhannagar, Kolkata 700064, INDIA*

## 1. Introduction

Through the last decade the experimental results from high energy nucleus nucleus collisions have proved to be extremely useful to characterize the thermodynamic properties of the strongly interacting system created in these collisions. Hydrodynamic simulations have been used effectively to describe the space-time evolution of the collisions at RHIC and LHC. Ideal hydrodynamics does not suffice to explain the elliptic flow ($v_2$) data of charged hadrons at RHIC and demands the inclusion of dissipative dynamics to describe the space-time evolution of the system. Since then the study of non-equilibrium processes have assumed a great deal of importance in the analyses of heavy ion collisions.

The dissipative processes within a system are quantified through the transport coefficients. The first order theories of relativistic dissipative fluid dynamics, in which the thermodynamic fluxes are linearly related to the thermodynamic forces involves the shear and bulk viscosities and thermal conductivity. However since the first order theory does not contain any time derivative of the corresponding fluxes, it results in the equations of motions that are parabolic in structure. This leads to the undesirable feature of acausality which results in infinite speed of the flows, making it impossible to estimate the relaxation times of the flows. This requires the use of a second order theory which produces hyperbolic equations of motions, leading to finite time scale over which the dissipative fluxes decay to their equilibrium values. These quantities which are called the relaxation times of the respective flows, give the measure of the time scale over which the energy-momentum or heat is being transported in a dissipative medium. In this paper we have evaluated the relaxation times of the viscous and the thermal flows for a hot interacting pion gas out of chemical equilibrium. The novel feature of these calculations is the use of the in-medium pion-pion scattering cross-section calculated using thermal field theory which is seen to affect the temperature dependence of the transport coefficients significantly. In the following sections some details of the calculation have been briefly described.

## 2. Evaluating of the relaxation times

The relativistic transport equation which has been solved using Grad's 14-moment method, takes the form

$$p^\mu \partial_\mu f_0 + f_0(1+f_0)p^\mu \partial_\mu \phi(x,p) = C[f], \quad (1)$$

where $f_0$ is the equilibrium distribution and $\phi(x,p)$ is a small deviation therefrom so that $f(x,p) = f_0(x,p) + f_0(1+f_0)[1+\phi(x,p)]$. Here $C[f]$ is the collision term which contains the amplitudes for various scattering processes occuring in the medium. In this method the trial function $\phi(x,p)$ is expanded as,

$$\phi = \sum_{s=0}^{2} A^s \tau^s - \sum_{s=0}^{1} B^s_\mu \tau^s \langle \Pi^\mu \rangle + C^0_{\mu\nu} \langle \Pi^\mu \Pi^\nu \rangle, \quad (2)$$

where the notation $\langle\rangle$ represents irreducible tensors of different ranks of $\Pi^\mu = p^\mu/T$ and $\tau = p \cdot u/T$. All the coefficient functions are symmetric and space-time dependent. After a long and tedious calculation the expressions

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[*] Electronic address: `utsav@vecc.gov.in`
[†] Electronic address: `sukanyam@vecc.gov.in`
[‡] Electronic address: `sourav@vecc.gov.in`

of relaxation times for bulk viscous flow, heat flow and shear viscous flow come out respectively as,

$$\begin{aligned}
\tau_\zeta =& \quad \frac{\zeta}{\alpha_2^2}\frac{1}{n^2}[\frac{a_3^3 - 2a_2a_3a_4 + a_1a_4^2}{a_2^2 - a_1a_3} + a_5] \\
\tau_\lambda =& \quad 3\frac{\lambda T}{\beta_1}\frac{1}{n^2}[3(b_3 - \frac{b_1b_2}{b_0}) - \frac{b_2}{h}] \\
\tau_\eta =& \quad 2\frac{\eta}{nT}[3 + \frac{z^2}{2h}]/\hat{h}. \qquad (3)
\end{aligned}$$

where $\zeta$, $\lambda$ and $\eta$ are the coefficients of bulk viscosity, thermal conductivity and shear viscosity respectively. The detailed forms of $a_i$ , $b_i$ etc. have been suppressed here for brevity.

## 3. Evaluating the transport coefficients

The first order transport coefficients i.e. $\zeta$, $\lambda$ and $\eta$ which go as input in the relaxation times estimated in the previous section are evaluated in the Chapman-Enskog approximation. The shear viscosity, bulk viscosity and thermal conductivity are respectively given by,

$$\eta = \frac{T}{10}\,\frac{\gamma_0^2}{c_{00}} \qquad \zeta = T\frac{\alpha_2^2}{a_{22}} \qquad \lambda = \frac{T}{3m_\pi}\,\frac{\beta_1^2}{b_{11}}.$$

The details of the calculation can be obtained from Refs. [1, 2].

## 4. The $\pi\pi$ cross-section in the medium

The $\pi\pi$ cross section is calculated phenomenologically by considering the scattering to proceed via $\rho$ and $\sigma$ meson exchange, using the effective Lagrangian,

$$\mathcal{L} = g_\rho \vec{\rho}^\mu . \vec{\pi} + \frac{1}{2} g_\sigma m_\sigma \vec{\pi}.\vec{\pi}\sigma \qquad (4)$$

Introducing decay widths of $\rho$ and $\sigma$ mesons in the corresponding s-channel processes the detailed matrix elements and the corresponding cross section are given in [1].

In order to obtain the $\pi\pi$ cross section in the medium the $\rho$ and the $\sigma$ widths appearing in the matrix elements are replaced with the corresponding in-medium ones. The effect of the medium on $\rho$ and $\sigma$ propagation is quantified through its self-energy, which at finite temperature can be evaluated using a tool from thermal field theory called real time formalism. The detail of this calculation is given in Ref. [3]. The self-energy function $\Pi$ which is related to the decay width by $k_0\Gamma(k) = -Im\Pi$, contains $\pi\pi$, $\pi\omega$, $\pi h_1$ and $\pi a_1$ loops. The mesons $\omega$, $h_1$ and $a_1$ all having substantial $3\pi$ and $\rho\pi$ decay widths, can be considered as a multi-pion contribution to the $\rho$ self-energy. The cross section obtained by using the in-medium $\rho$ propagator suffers a small suppression of the peak for $\pi\pi$ loop and a larger effect for $\pi$-meson loop.

## 5. Results

We have estimated the temperature dependence of the relaxation times of the respective flows incorporating the finite temperature medium effects with different values of pion chemical potential. We have used a temperature dependent chemical potential which increases with decreasing temperature using the fact of early chemical freeze out of pion gas in heavy ion collisions. The corresponding results show significant modifications of the temperature dependence of those relaxation times with the in medium cross sections compared to vacuum ones. The temperature dependent pion chemical potential also affects the results in a significant way.

## References

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[3] S. Mitra, S.Ghosh and S. Sarkar, Phys. Rev. C **85** (2012) 064917.